Short Papers

Two-Dimensional Localization: Low-Rank Matrix Completion With Random Sampling in Massive MIMO System

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***Abstract*—In this paper, random sampling is considered for direction-of-arrival (DOA) estimation with reduced hardware complexity in massive multiple-input–multiple-output (MIMO) systems. The resulting problem is that the accuracy of the existing DOA estimators will significantly degrade with the availability of only a small subset of data entries as the low date rate is employed to reduce the system power consumption. To address that, an efficient approach based on the variant of matrix factorization is devised to complete the underlying data matrix. As a result, the nominal azimuth and elevation DOAs with the corresponding angular spreads are estimated from the underlying data matrix. Numerical results demonstrate that even in the case of missing entries, the proposed method is superior to the existing approaches with full data.**

***Index Terms*—Angular spread, low-rank matrix completion, massive multiple-input–multiple-out (MIMO) system, random sampling scheme, two-dimensional direction-of-arrival (2-D DOA).**

I. Introduction

With the almost double increase of mobile data traffic in every year, massive multiple-input–multiple-output (MIMO) technique becomes an emerging physical layer candidate for future wireless communication systems due to high data rate, enhanced link reliability and potential power saving [1], [2]. As a base station (BS) will rely on the uplink signals to figure out the channel knowledge to perform MIMO beamforming, direction-of-arrival (DOA) at the BS plays an important role in massive MIMO system, especially for the existence of angular spreads.

Numerous DOA estimation approaches [3]–[5] have been proposed for the distributed sources model with multipath transmission in cellular wireless systems. One typical representative of subspace-based method, the estimation of signal parameters via rotational invariance technique (ESPRIT), enjoys high resolution and reduced computational burden for DOA estimation. In [4], based on the main difficulty of ESPRIT that with $a\ priori$ information of the effective dimension of the pseudosignal subspace, it employs the subspace principle without eigendecomposition of sample covariance matrix to estimate DOAs. In [5], by dividing the uniform rectangular array (URA) into three subarrays, 2-D nominal DOAs and angular spreads are obtained from the combination of eigenvalues on the subarrays.

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Nevertheless, the large array aperture and limited hardware cost will generally result in serious challenges for parameter estimation and real-time signal processing. Owing to the low-rank matrix completion theory [6] that can support the design of new sampling scheme, the random sampling scheme [7] is introduced in MIMO system for 2-D localization, which will significantly reduce the hardware complexity with a $randomly$ chosen fraction of training data.

Motivated by that to save the power consumption especially for the systems with battery limit in practical applications, the concept of random sampling scheme is introduced for 2-D DOA estimation in massive MIMO system, which results in the problem of matrix completion. Then, an efficient alternating direction method of multipliers (ADMM)-based matrix completion algorithm is developed on the basis of the matrix factorization framework. Borrowed the idea from [4], DOAs with the corresponding angular spreads are finally estimated from the reconstructed data matrix via a subspace-based method.

II. Data Model

Consider a massive MIMO system employing a URA with $M = M_x M_y$ sensors, where M_x and M_y are the number of elements in the X-direction and Y-direction, respectively. Without loss of generality, the origin is chosen as the reference point of the array. Assume that there are K incoherently distributed sources in the same frequency band, transmitted by user terminals (UTs) impinge on the BSs. In the presence of additive Gaussian noise, the output of receiver array at time instant t is modeled as [8]

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t),\ t = 1, \ldots, N \tag{1}$$

where N is the number of snapshots, $\mathbf{s}(t)$ is the complex-valued transmitted signal transmitted by K UTs, and the steering matrix $\mathbf{A} \in \mathbb{C}^{M\times K}$ is composed of the array manifold $\mathbf{a}(\phi_\ell, \theta_\ell) = \mathbf{a}_x(\phi_\ell, \theta_\ell) \otimes \mathbf{a}_y(\phi_\ell, \theta_\ell), \ell = 1, \ldots, K$, where the array response with respect to elevation DOA θ and azimuth DOA ϕ are $\mathbf{a}_x(\phi_\ell, \theta_\ell) = e^{j2\frac{\pi}{\lambda}(m_x-1)d_x \sin(\theta_\ell)\cos(\phi_\ell)}, m_x = 1, \ldots, M_x$ and $\mathbf{a}_y(\phi_\ell, \theta_\ell) = e^{j2\frac{\pi}{\lambda}(m_y-1)d_y \sin(\theta_\ell)\sin(\phi_\ell)}, m_y = 1, \ldots, M_y$, respectively. d_x and d_y stand for the element spacing in the X-direction and Y-direction, respectively. In the massive MIMO system, when angular spreads are not wide enough, the system performance will seriously degrade. Therefore, an extra beamforming approach is required to devise for achieving directional antenna gain. Nevertheless, the performance of the beamforming approaches often closely relies on the accuracy of the estimated angular parameters. For instance, estimation errors at 0.1° and 0.04° will cause 20 and 3 dB reductions of the output signal-to-noise ratio (SNR), respectively.

In the sequel, similar with the off-grid problem [9], the elevation and azimuth DOAs are respectively remodified with angular spreads

$$\bar{\phi}_\ell = \phi_\ell + \Delta_{\phi,\ell},\ \bar{\theta}_\ell = \theta_\ell + \Delta_{\theta,\ell} \tag{2}$$

in which $\Delta_{\phi,\ell}$ and $\Delta_{\theta,\ell}$ are the corresponding random angular deviations with zero-mean and standard deviations σ_{ϕ_ℓ} and σ_{θ_ℓ}, respectively. Taking the first-order Taylor series expansion of $\mathbf{a}(\phi_\ell, \theta_\ell)$ around the nominal DOAs ϕ_ℓ and θ_ℓ, we have

$$\mathbf{a}(\bar{\phi}_\ell, \bar{\theta}_\ell) = \mathbf{a}(\phi_\ell + \Delta_{\phi,\ell}, \phi_\ell + \Delta_{\phi,\ell})$$
$$= \mathbf{a}(\phi_\ell, \theta_\ell) + \frac{\partial \mathbf{a}(\bar{\phi}_\ell, \bar{\theta}_\ell)}{\partial \phi_\ell}\Delta_{\phi,\ell} + \frac{\partial \mathbf{a}(\bar{\phi}_\ell, \bar{\theta}_\ell)}{\partial \theta_\ell}\Delta_{\theta,\ell}.$$

Therefore, (1) is rewritten as

$$\bar{\mathbf{x}}(t) = \bar{\mathbf{A}}\mathbf{c}(t) + \mathbf{n}(t) \tag{3}$$

in which the novel steering matrix is

$$\bar{\mathbf{A}} = \left[\mathbf{a}(\phi_1, \theta_1), \ldots, \mathbf{a}(\phi_K, \theta_K), \frac{\partial \mathbf{a}(\bar{\phi}_1, \bar{\theta}_1)}{\partial \phi_1}, \ldots, \frac{\partial \mathbf{a}(\bar{\phi}_K, \bar{\theta}_K)}{\partial \phi_K}, \frac{\partial \mathbf{a}(\bar{\phi}_1, \bar{\theta}_1)}{\partial \theta_1}, \ldots, \frac{\partial \mathbf{a}(\bar{\phi}_K, \bar{\theta}_K)}{\partial \theta_K}\right]$$

and $\mathbf{c}(t) = [c_{1,1}, \ldots, c_{K,1}, c_{1,2}, \ldots, c_{K,2}, c_{1,3}, \ldots, c_{K,3}]$ with $c_{\ell,1}(t) = s_k(t)\sum_{i=1}^{N_\ell}\gamma_{\ell,i}$, $c_{\ell,2}(t) = s_k(t)\sum_{i=1}^{N_\ell}\gamma_{\ell,i}\Delta_{\phi,\ell}$, $c_{\ell,3}(t) = s_k(t)\sum_{i=1}^{N_\ell}\gamma_{\ell,i}\Delta_{\theta,\ell}$. N_ℓ is the number of multipaths and $\gamma_{\ell,i}$ denotes the complex-valued path gain.

Since the signal and noise are uncorrelated, the covariance matrix of the received signal can be expressed as

$$\mathbf{R} = \mathbb{E}\{\bar{\mathbf{x}}(t)\bar{\mathbf{x}}^H(t)\} = \mathbf{A}\mathbf{\Sigma}_c\mathbf{A}^H + \sigma^2\mathbf{I} \tag{4}$$

where $\mathbf{\Sigma}_c = \mathbb{E}\{\mathbf{c}(t)\mathbf{c}^H(t)\}$. From (4), we observe that $\mathbf{\Sigma}_c$ can be estimated by $\mathbf{\Sigma}_c = \mathbf{A}^\dagger(\mathbf{R} - \sigma^2\mathbf{I})(\mathbf{A}^H)^\dagger$, where † denotes the pseudoinverse operator. Hence, the angular spreads σ_ϕ and σ_θ can be computed by

$$\sigma_\phi = \sqrt{\frac{[\mathbf{\Sigma}_c]_{2K+\ell,2K+\ell}}{[\mathbf{\Sigma}_c]_{\ell,\ell}}} \qquad \sigma_\theta = \sqrt{\frac{[\mathbf{\Sigma}_c]_{K+\ell,K+\ell}}{[\mathbf{\Sigma}_c]_{\ell,\ell}}}. \tag{5}$$

Obviously, the accuracy of the estimated angular spreads σ_ϕ, σ_θ depends on the estimated nominal DOAs ϕ, θ, respectively. The job is to estimate ϕ, θ, σ_ϕ and σ_θ in massive MIMO system.

III. Proposed Method With Random Sampling

The requirement of a huge number of front-end units seriously challenge on the design of mobile system, with the problem of high hardware complexity. This is because the data sampling rate is directly related to the power consumption. It is well known that to capture signals with reasonable quality, it requires a continuous sampling rate no less than the maximum signal frequency according to the celebrated Shannon's theorem. For example, for a state-of-the-art radio that consumes about 4nJ/bit and assuming samples of a 16-b resolution, this converts to about 500 μW power consumption, which is prohibitive as it will drain a typical 1.5 V 200 mAh silver coin cell battery in merely a month. More importantly, the measurements are technically prohibitive to collect and transmit all but a *randomly* chosen fraction $p \in (0, 1)$ of the training data, due to high experimental and uniform sampling costs, hardware limitation, or other inevitable reasons in some circumstances, e.g., MIMO-MC radar [7], massive MIMO system [10], etc.

In this paper, we consider the problem of identifying DOAs contained in spectrally sparse signals—namely, a superposition of UTs from a random set of partial nonuniform samples (referred to here as the missing/incomplete data case). It is well known that the DOAs can be estimated by the classical spectrum estimation approaches, e.g., ESPRIT, when full observations are available. Nevertheless, the limitation is that they are not compatible with the random sampling or compression protocols that can be used to reduce the front-end sampling burden [11]. Hence, with the random sampling scheme, we need to devise a method to recover the full data matrix based on knowledge of a small subset of its entries.

Collecting all snapshots, we obtain the received data matrix $\mathbf{Y} = \{\bar{\mathbf{x}}\}_{t=1}^N \in \mathbb{C}^{M\times N}$ as the compact version of (3). The low-rank matrix completion theory [6] can support the design of new sampling scheme, enabling significant reduction of the volume of data required for accurate localization. Using the matrix completion technique, 2-D DOA estimation problem with random sampling can be expressed as the following optimization problem [6], [7]:

$$\min ||\mathbf{X}||_* + \frac{\lambda}{2}||\mathbf{Y}_\Omega - \mathbf{X}_\Omega||_F^2 \tag{6}$$

where $\lambda > 0$ denotes the penalty parameter, $||\cdot||_*$ and $||\cdot||_F$ denote the nuclear and Frobenius norms, respectively. The set Ω contains all matrix coordinates corresponding to the observed entries in $\mathbf{Y}$, and Frobenius norm error term is robust against the additive Gaussian noise. The projection matrix is defined as

$$[\mathbf{X}_\Omega]_{ij} = \begin{cases} \mathbf{X}_{ij}, & (i,j) \in \Omega \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

Based on the rationale of the matrix factorization ($\mathbf{X} = \mathbf{U}\mathbf{V}^H$), the problem (6) is equivalently written as

$$\min_{\mathbf{U},\mathbf{V}} ||\mathbf{Y}_\Omega - (\mathbf{U}\mathbf{V}^H)_\Omega||_F^2 \tag{8}$$

where $\mathbf{U} \in \mathbb{C}^{M\times r}$, $\mathbf{V}^H \in \mathbb{C}^{r\times N}$ and $r << \min(M, N)$ equals to the number of UTs. Due to the fact that

$$\sum_{\ell=1}^{r}\sigma_\ell(\mathbf{U}\mathbf{V}^H) \le \sum_{\ell=1}^{r}\sigma_\ell(\mathbf{U})\sigma_\ell(\mathbf{V}^H) \le \left(\sum_{\ell=1}^{r}\sigma_\ell^a(\mathbf{U})\right)^{\frac{1}{a}} \times \left(\sum_{\ell=1}^{r}\sigma_\ell^a(\mathbf{V}^H)\right)^{\frac{1}{b}} \le \frac{1}{a}\left(\sum_{\ell=1}^{r}\sigma_\ell^a(\mathbf{U})\right) + \frac{1}{b}\left(\sum_{\ell=1}^{r}\sigma_\ell^a(\mathbf{V}^H)\right) \tag{9}$$

with $\sigma_\ell(\cdot)$ denoting the singular values of matrix. The reasons for the existence of (9) are analyzed as the following.

1) The first inequality holds because of

$$|\mathrm{trace}(\mathbf{U}\mathbf{V}^H)| \le \sum_{\ell=1}^{\min\{M,N\}}\sigma_\ell(\mathbf{U})\sigma_\ell(\mathbf{V}^H)$$

for any matrices $\mathbf{U}$ and $\mathbf{V}$, where $\mathrm{trace}(\cdot)$ denotes the trace function. It has been proved in 3 [12, Lemma 3].

1) The second inequality holds due to Holder's inequality with $(1/a) + (1/b) = 1$.
2) The third inequality holds because of Jensen's inequality by taking the logarithm on both sides [13].

Hence, (8) is converted to

$$\min_{\mathbf{U},\mathbf{V}} \frac{1}{a}||\mathbf{U}^a||_* + \frac{1}{b}||\mathbf{V}^b||_* + \frac{\lambda}{2}||\mathbf{Y}_\Omega - (\mathbf{U}\mathbf{V}^H)_\Omega||_F^2. \tag{10}$$

ADMM, as a powerful optimization framework, is suitable for large-scale problems arising in machine learning and signal processing. Since $\mathbf{U}$ and $\mathbf{V}$ are coupled with each other, two auxiliary variables are introduced, which results in the following optimization problem:

$$\min_{\mathbf{U},\mathbf{V},\hat{\mathbf{U}},\hat{\mathbf{V}}} \frac{1}{2}\left(\sum_{i=1}^{r}\sigma_i^2(\mathbf{U})\right)+\frac{1}{2}\left(\sum_{i=1}^{r}\sigma_i^2(\mathbf{V})\right) + \frac{\lambda}{2}||\mathbf{Y}_\Omega-(\hat{\mathbf{U}}\hat{\mathbf{V}}^H)_\Omega||_F^2 \quad \text{s.t. } \hat{\mathbf{U}}=\mathbf{U}, \hat{\mathbf{V}}=\mathbf{V} \tag{11}$$

where a and b are simplified to set at 2. Therefore, the Lagrangian function of (11) is

$$\mathcal{L}=\frac{1}{2}\left(\sum_{i=1}^{r}\sigma_i^2(\mathbf{U})\right)+\frac{1}{2}\left(\sum_{i=1}^{r}\sigma_i^2(\mathbf{V})\right) + \frac{\lambda}{2}||\mathbf{Y}_\Omega-(\hat{\mathbf{U}}\hat{\mathbf{V}}^H)_\Omega||_F^2+\langle\mathbf{\Gamma},\hat{\mathbf{U}}-\mathbf{U}\rangle + \langle\mathbf{\Upsilon},\hat{\mathbf{V}}-\mathbf{V}\rangle+\frac{\eta}{2}||\hat{\mathbf{U}}-\mathbf{U}||_F^2+\frac{\eta}{2}||\hat{\mathbf{V}}-\mathbf{V}||_F^2 \tag{12}$$

where the penalty parameter η is a positive value, $\mathbf{\Gamma}_\Omega$ and $\mathbf{\Upsilon}_\Omega$ contain $|\Omega|$ Lagrange multipliers. $\langle\cdot,\cdot\rangle$ denotes the inner product of two matrices.

Step 1: Update $\hat{\mathbf{U}}$ and $\hat{\mathbf{V}}$

$$\min_{\hat{\mathbf{U}}} \frac{\lambda}{2}||\mathbf{Y}_\Omega-(\hat{\mathbf{U}}\hat{\mathbf{V}}^H)_\Omega||_F^2+\left\langle\mathbf{\Gamma},\hat{\mathbf{U}}-\mathbf{U}\right\rangle+\frac{\eta}{2}||\hat{\mathbf{U}}-\mathbf{U}||_F^2 \tag{13}$$

$$\min_{\hat{\mathbf{V}}} \frac{\lambda}{2}||\mathbf{Y}_\Omega-(\hat{\mathbf{U}}\hat{\mathbf{V}}^H)_\Omega||_F^2+\left\langle\mathbf{\Upsilon},\hat{\mathbf{V}}-\mathbf{V}\right\rangle+\frac{\eta}{2}||\hat{\mathbf{V}}-\mathbf{V}||_F^2. \tag{14}$$

They can be further rewritten as

$$\min_{\hat{\mathbf{U}}} \frac{\lambda}{2}||\mathbf{Y}_\Omega-(\hat{\mathbf{U}}\hat{\mathbf{V}}^H)_\Omega||_F^2+\frac{\eta}{2}||\hat{\mathbf{U}}-\mathbf{U}+\frac{\mathbf{\Gamma}}{\eta}||_F^2 \tag{15}$$

$$\min_{\hat{\mathbf{V}}} \frac{\lambda}{2}||\mathbf{Y}_\Omega-(\hat{\mathbf{U}}\hat{\mathbf{V}}^H)_\Omega||_F^2+\frac{\eta}{2}||\hat{\mathbf{V}}-\mathbf{V}+\frac{\mathbf{\Upsilon}}{\eta}||_F^2. \tag{16}$$

Both of them are least squares problems and can easily obtain their closed-form solutions

$$\begin{cases}\hat{\mathbf{U}}^{t+1} &= \left(\eta\mathbf{U}-\mathbf{\Gamma}+\lambda\mathbf{Y}_\Omega\hat{\mathbf{V}}\right)\left(\eta\mathbf{I}_r+\lambda\hat{\mathbf{V}}^H\hat{\mathbf{V}}\right)^{-1}\\ \hat{\mathbf{V}}^{t+1} &= \left(\eta\mathbf{V}-\mathbf{\Upsilon}+\lambda\mathbf{Y}_\Omega^H\hat{\mathbf{U}}\right)\left(\eta\mathbf{I}_r+\lambda\hat{\mathbf{U}}^H\hat{\mathbf{U}}\right)^{-1}.\end{cases} \tag{17}$$

Step 2: Update $\mathbf{U}$ and $\mathbf{V}$

$$\min_{\mathbf{U}} \frac{1}{2}\left(\sum_{i=1}^{r}\sigma_i^2(\mathbf{U})\right)+\frac{\eta}{2}||\hat{\mathbf{U}}-\mathbf{U}+\frac{\mathbf{\Gamma}}{\eta}||_F^2 \tag{18}$$

$$\min_{\mathbf{V}} \frac{1}{2}\left(\sum_{i=1}^{r}\sigma_i^2(\mathbf{V})\right)+\frac{\eta}{2}||\hat{\mathbf{V}}-\mathbf{V}+\frac{\mathbf{\Upsilon}}{\eta}||_F^2. \tag{19}$$

They are nuclear norm minimization, which can be solved by singular value thresholding (SVT) [14]

$$\begin{cases}\mathbf{U}^{t+1} &= \mathbf{L}_U\,\mathrm{Diag}\left(\sigma(\hat{\mathbf{U}}+\frac{\mathbf{\Gamma}}{\eta})\right)\mathbf{R}_U^H\\ \mathbf{V}^{t+1} &= \mathbf{L}_V\,\mathrm{Diag}\left(\sigma(\hat{\mathbf{V}}+\frac{\mathbf{\Upsilon}}{\eta})\right)\mathbf{R}_V^H\end{cases} \tag{20}$$

where $\mathbf{L}_U\mathbf{\Sigma}_U\mathbf{R}_U^H=\hat{\mathbf{U}}+\frac{\mathbf{\Gamma}}{\eta}$, $\mathbf{L}_V\mathbf{\Sigma}_V\mathbf{R}_V^H=\hat{\mathbf{V}}+\frac{\mathbf{\Upsilon}}{\eta}$, and $\mathrm{Diag}(\cdot)$ returns a square diagonal matrix.

Step 3: Update $\mathbf{\Gamma}$ and $\mathbf{\Upsilon}$

$$\mathbf{\Gamma}^{t+1}=\mathbf{\Gamma}+\eta(\hat{\mathbf{U}}-\mathbf{U}),\ \mathbf{\Upsilon}^{t+1}=\mathbf{\Upsilon}+\eta(\hat{\mathbf{V}}-\mathbf{V}) \tag{21}$$

where $\eta=\min\{\rho\eta,\eta_{\max}\}$, $\rho>1$. The target matrix $\hat{\mathbf{X}}$ is computed by $\hat{\mathbf{X}}=\mathbf{U}\mathbf{V}^H$ after determining $\mathbf{U}$ and $\mathbf{V}$. The computational complexity of the proposed method is $\mathcal{O}(KMNr)$. The procedure of the proposed method is summarized in Algorithm 1. Although the convergence theory has been well established for a variety of ADMM variants in [15], including multiblock ADMM, the corresponding results are not applicable to our problem with missing entries, and this challenging problem will remains an open problem for future research. Next, we briefly analyze the reasonability of solution $\hat{\mathbf{X}}=\mathbf{X}^*$. Assume that

Algorithm 1: The Proposed Method.

Require: $\boldsymbol{Y}_\Omega$, $\rho=1.1+0.5p$, $\eta_{\max}=3$, $\lambda=0.5$, and p
  Initialize: random variables $\mathbf{U}$, $\mathbf{V}$, $\mathbf{\Gamma}$ and $\mathbf{\Upsilon}$, $\eta=2/(10||\mathbf{Y}_\Omega||_2)$
  for $t=1,2,\cdots$ **do**
    1) Update $\hat{\mathbf{U}}^{t+1}$ and $\hat{\mathbf{V}}^{t+1}$ by (17)
    2) Update $\mathbf{U}^{t+1}$ and $\mathbf{V}^{t+1}$ by (20)
    3) Update $\mathbf{\Gamma}$ and $\mathbf{\Upsilon}$ by (21)
    Stop if convergent.
  end for
Ensure: $\hat{\mathbf{X}}=\mathbf{U}\mathbf{V}^H$

$$\hat{\mathbf{X}}=\arg\min_{\mathbf{X}}||\mathbf{X}||_*,\ \text{s.t. } \mathbf{X}=\mathbf{X}^* \tag{22}$$

in which $\mathbf{X}^*$ is the data matrix in noiseless case. Trivially, this problem has a solution, that is, $\hat{\mathbf{X}}=\mathbf{X}^*$. Suppose that the corresponding dual feasible point of (22) is $\mathbf{Q}$, which is a solution of the dual problem via the Lagrange function of (22). The Lagrangian function is expressed as

$$\begin{aligned}\mathcal{L}(\mathbf{X},\mathbf{Q}) &= ||\mathbf{X}||_*+<\mathbf{X}^*-\mathbf{X},\mathbf{Q}>_\mathbb{R}\\ &= ||\mathbf{X}||_*-<\mathbf{X},\mathbf{Q}>_\mathbb{R}+<\mathbf{X}^*,\mathbf{Q}>_\mathbb{R}\end{aligned} \tag{23}$$

with $\mathbf{Q}$ being the dual variable, where $<\cdot,\cdot>_\mathbb{R}$ stands for the real inner product of two matrices. Taking the subgradient of $\mathcal{L}(\mathbf{X},\mathbf{Q})$ with respect to $\mathbf{X}$, that is, $\frac{\partial\mathcal{L}(\mathbf{X},\mathbf{Q})}{\partial\mathbf{X}}=\partial||\mathbf{X}||_*-\mathbf{Q}$, where the subdifferential of nuclear norm is written as [14]

$$\begin{aligned}&\partial||\mathbf{X}||_*=\partial||\mathbf{X}^*||_*\\ &=\left\{\mathbf{Z}:\mathbf{Z}=\mathbf{U}_{\mathbf{X}^*}\mathbf{V}_{\mathbf{X}^*}^H+\mathbf{W},\mathbf{U}_{\mathbf{X}^*}^H\mathbf{W}=\mathbf{0},\mathbf{W}\mathbf{V}_{\mathbf{X}^*}=\mathbf{0},||\mathbf{W}||\le 1\right\}.\end{aligned}$$

Thus, $\mathbf{X}^*=\mathbf{U}_{\mathbf{X}^*}\mathbf{S}_{\mathbf{X}^*}\mathbf{V}_{\mathbf{X}^*}^H$ is a truncated SVD of $\mathbf{X}^*$ with $\mathbf{U}_{\mathbf{X}^*}\in\mathbb{C}^{M\times K}$, $\mathbf{S}_{\mathbf{X}^*}\in\mathbb{C}^{K\times K}$ and $\mathbf{V}_{\mathbf{X}^*}\in\mathbb{C}^{N\times K}$. Due to the zero-gradient condition in the Karush–Kuhn–Tucker conditions, let $\frac{\partial\mathcal{L}(\mathbf{X},\mathbf{Q})}{\partial\mathbf{X}}=\mathbf{0}$, we can find $\mathbf{Q}=\mathbf{U}_{\mathbf{X}^*}\mathbf{V}_{\mathbf{X}^*}^H$ by choosing $\mathbf{W}=\mathbf{0}$.

Finally, DOAs and angular spreads are obtained by utilizing the estimator in [4] on target matrix $\mathbf{X}$.

IV. Simulation Results

We evaluate the performance of the proposed method in massive MIMO system for sources localization, where the emitting sources are binary phase shift keying signals. The number of multipaths, snapshots, and BSs are $N_\ell=50$, $N=500$, and $M_x=M_y=10$, respectively, and the angular spreads are set at $\sigma_{\theta_\ell}=\sigma_{\phi_\ell}=1°$. We consider $K=5$ UTs, and the elevation and azimuth DOAs are chosen as $(\boldsymbol{\theta},\boldsymbol{\phi})=\{(10°, 30°), (30°, 50°), (50°, 40°), (110°, 80°), (130°, 70°)\}$. For the random sampling, the cases of $p=0.8$ and $p=0.9$ are respectively considered.

The root-mean-square error (RMSE) performance versus the number of UTs and SNR are investigated, respectively, as shown in Figs. 1 and 2. The results of several algorithms [3]–[5] are also included

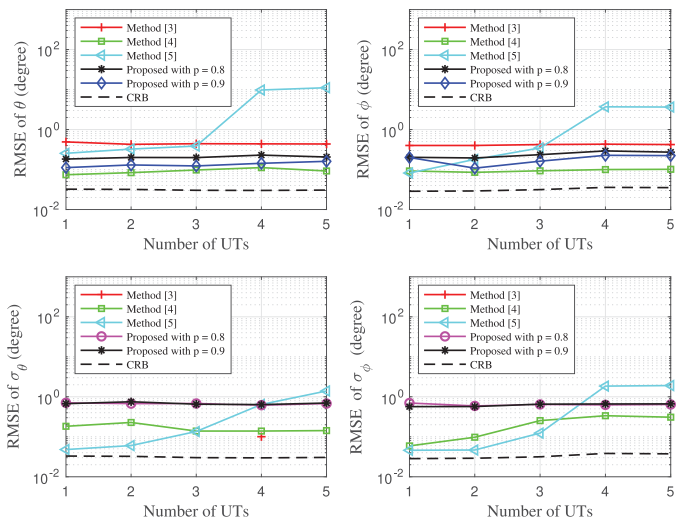

Fig. 1. RMSE versus the number of UTs.

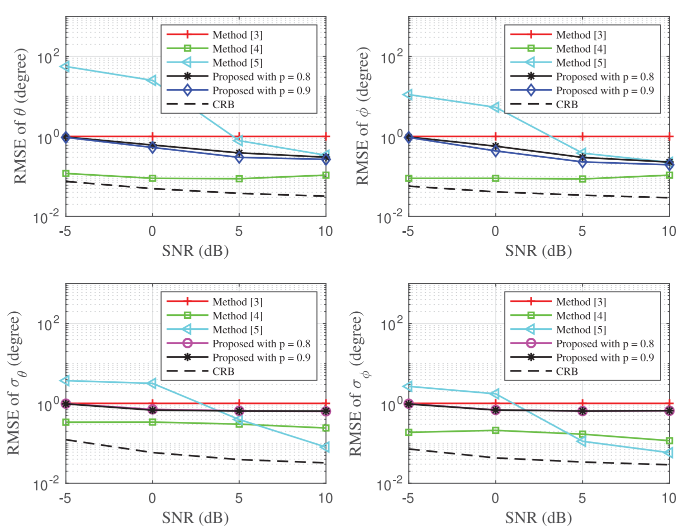

Fig. 2. RMSE versus SNR.

for comparison. It is observed that the performance of the proposed method with $p = 0.9$ is slightly less than that of the baseline [4], which is reasonable because of 10% missing entries, however, it can greatly reduce the hardware complexity. Thus, in the case of $p = 0.8$, it still performs better than other compared approaches at full data since the proposed method is more robust against noise, except for angular spreads results. Besides, we note that the RMSE of the proposed method increase slowly as the number of the UTs increases. This is because the nominal DOAs of the UTs are only estimated by searching around the true values in the proposed method, which is based on the assumption that the coarse estimates of the nominal DOAs have been obtained.

V. Conclusion

In this paper, an efficient ADMM-based matrix completion approach is proposed to complete the underlying data matrix from a subset of samples based on random sampling scheme, in order to save the hardware cost. On the basis of the recovered data matrix, DOAs and their angular spreads are estimated by subspace-based method. Simulation results have shown the effectiveness of the proposed method.

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