Compressed Sensing-Based Angle Estimation for Noncircular Sources in MIMO Radar

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Compressed sensing-based angle estimation for noncircular sources in MIMO radar

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***Abstract*—In this paper, we consider the problem of applying compressed sensing (CS) theory to angle estimation for noncircular sources in monostatic multiple-input multiple-output (MIMO) radar, and propose an angle estimation algorithm based on extended matrix compressed sensing. Firstly, a reduced-dimensional matrix is employed to transform the data matrix into a low dimensional one. Then the properties of noncircular signals are utilized to construct an extended matrix from the received data. Finally, the dictionary can be conducted to apply Orthogonal Matching Pursuit (OMP) for angle estimation. The angle estimation performance of the proposed algorithm is better than that of estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm and reduced-dimension ESPRIT (RD-ESPRIT) algorithm, and the proposed method requires no knowledge of the noise. The simulation results verify the effectiveness of the algorithm.**

Keywords-monostatic radar; noncircular signal; OMP; angle estimation

I. INTRODUCTION

Multiple-input multiple-output (MIMO) radars use multiple antennas to simultaneously emit distinct waveforms and utilize multiple antennas to receive the reflected signals, which is widely developed based on MIMO communication theory. As a new radar system, MIMO radar has been researched for many years, because of its lots of potential advantages over the conventional phased-array radar, such as higher angle estimation accuracy, better parameter identifiability, more degree of freedom (DOF) and so on [1-6]. Angle estimation of targets from the received signals corrupted by noise is an important aspect in MIMO radar. Nowadays, a variety of algorithms for angle estimation are presented, which include estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm [7], Capon algorithm [8], multiple signal classification (MUSIC) algorithm [9], parallel factor analysis (PARAFAC) algorithms [10] and adaptive parallel factor analysis algorithms [11]. All of algorithms can improve the angle estimation performance with the multiple snapshots instead of one snapshot.

Compressed sensing has attracted a variety of attentions in the recent years, which it is applied to dealing with a lot of problems, including medical imaging [12], radar imaging and detection [13], Wireless communication [14], Optical imaging [15] and so on. The l_1-SVD method in [16] uses l_1 norm to reduce sensitivity to noise and to enhance the sparsity of solution, meanwhile, they focus the problem on the subspace domain instead of the data domain. In [17], the proposed algorithm applies sparse representation scheme for angle estimation and use weight l_1-SVD constraint minimization to enforce the sparsity. However, both the method in [16] and [17] have a common problem, that is the choice of the regularization parameter, so a prior information of the target number or the noise should be known.

In this paper, we address the problem of applying compressed sensing (CS) theory to angle estimation for noncircular sources in monostatic multiple-input multiple-output (MIMO) radar. Firstly, an extended matrix is constructed by using the properties of noncircular signals, whose elements are twice as many as the monostatic MIMO radar virtual array with diverse elements. Then, according to the relationship between the direction matrix and signal subspace, the signal subspace is realigned to apply the orthogonal matching pursuit (OMP) for angle estimation. The angle estimation performance of the proposed algorithm is better than that of RD-ESPRIT algorithm, and ESPRIT algorithm. Furthermore, the proposed algorithm does not choose the regularization parameter, so it requires no knowledge of the noise. The simulation results verify the effectiveness of the algorithm.

Notation: $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$ and $(\cdot)^*$ denote the transpose, Hermitian transpose, inverse, complex conjugation without transposition, respectively. $vec(\cdot)$ denotes a matrix operation the stacks the columns of a matrix under each other to form a new vector, $\otimes$ denotes the Kronecker operator, $diag(\cdot)$ denotes the diagonalization operation.

II. DATA MODEL

Consider a standard direction-of-arrival (DOA) estimation scenario with M elements using for both transmit and receive sensor arrays (Fig.1). The transmit arrays and receive arrays are assumed to be closely located

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so that a target located in the far-field can be seen by both of them at the same angle. All antennas are uniform linear arrays (ULAs) omnidirectional. And all elements transmit orthogonal noncircular signals. Assumed that additional phase shift caused by Doppler frequency does not change with time and has no influence on the orthogonal signal. There are K uncorrelated narrowband far-field targets and the signals arrived at the receive array through reflections of the targets can be described as

$$\mathbf{X} = \sum_{k=1}^{K} \beta_k \mathbf{a}(\theta_k)\mathbf{a}^T(\theta_k)\mathbf{S} + \mathbf{W} \tag{1}$$

Where θ_k is the direction of arrival (DOA) of the kth target with respect to the transmit array normal or the receive array normal; f_k is the Doppler frequency of the kth target and β_k is the real-valued amplitude of the kth target, $\mathbf{S} \in \mathbb{C}^{M\times L}$ is the orthogonal and noncircular transmit signal matrix, and $\mathbf{W} \in \mathbb{C}^{M\times L}$ is the complex Gaussian white noise vector with zeros mean and covariance matrix $\sigma^2\mathbf{I}$, $\mathbf{a}(\theta_k) = [1 \quad e^{j\pi\sin\theta_k} \quad \cdots \quad e^{j\pi(\mathrm{M}-1)\sin\theta_k}]^T \in \mathbb{C}^{M\times 1}$ is the steering vector of the kth target.

Owing to the orthogonality property of the transmitted signals, after the matched filtering, the received data can be written as

$$\bar{\mathbf{X}} = \sum_{k=1}^{K} \beta_k \mathbf{a}(\theta_k)\mathbf{a}^T(\theta_k) + \bar{\mathbf{W}} \tag{2}$$

Where $\bar{\mathbf{W}} = \mathbf{W}\mathbf{S}^H$ is the noise matrix after matched filters.

Stacking each succeeding column of $\bar{\mathbf{X}} \in \mathbb{C}^{M\times M}$, we obtain the $M^2 \times 1$ virtual data vector

$$\mathbf{Y} = vec(\bar{\mathbf{X}}) = \sum_{k=1}^{K} \beta_k \mathbf{b}(\theta_k) + \mathbf{N} \tag{3}$$

Where $\mathbf{b}(\theta_k) = \boldsymbol{\alpha}(\theta_k) \otimes \boldsymbol{\alpha}(\theta_k) \in \mathbb{C}^{M^2\times 1}$ is the transmit-receive steering vector, $\mathbf{N} = vec(\bar{\mathbf{W}}) \in \mathbb{C}^{M^2\times 1}$ is a zero-mean complex Gaussian white noise vector with covariance matrix $\sigma^2\mathbf{I}_{M^2\times 1}$.

Assumed there is a length P periodic pulse train to temporally sample the signal environment, $\mathbf{H}(p) = diag[\beta_1(p), \cdots, \beta_K(p)]$ stands for the reflected noncircular signal of targets after matching filters, which satisfies with $\mathbf{H}(p) = \mathbf{H}^*(p)$ according to the property of noncircular signal, then the received data in Eq.(3) can be expressed as

$$\mathbf{Y}(p) = \mathbf{B}(\theta)\mathbf{H}(p) + \mathbf{N}(p), p = 1, 2, \ldots, P \tag{4}$$

Where $\mathbf{B}(\theta) = [\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \cdots, \mathbf{b}(\theta_K)]$, $\mathbf{N}(p)$ is the noise vector of the pth pulse.

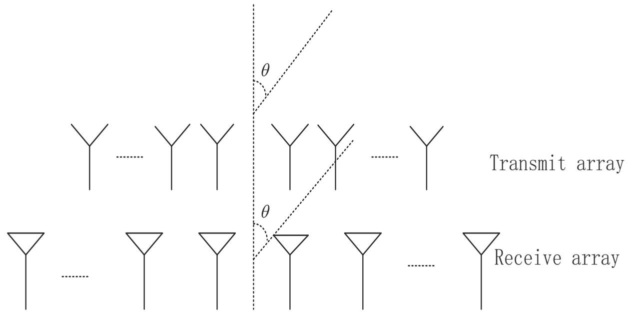

Fig. 1. The configuration of monostatic MIMO radar

III. Compressed Sensing-Based Angle Estimation for Noncircular Source in MIMO Radar

A. Reduced-Dimension Transformation

Owing to the transmit-receive steering vector $\mathbf{b}(\theta_k) = \boldsymbol{\alpha}(\theta_k) \otimes \boldsymbol{\alpha}(\theta_k) \in \mathbb{C}^{M^2\times 1}$ can be expressed as

$$\mathbf{b}(\theta_k) = \boldsymbol{\alpha}(\theta_k) \otimes \boldsymbol{\alpha}(\theta_k) = \mathbf{G}\mathbf{f}(\theta_k) \tag{5}$$

Where $\mathbf{f}(\theta_k) = [1, \mathrm{e}^{-j\pi\sin\theta_k}, \cdots, \mathrm{e}^{-j\pi(2\mathrm{M}-2)\sin\theta_k}]^T$ is the steering vector of the virtual uniform linear array.

$$\mathbf{G} = \begin{bmatrix} \left.\begin{matrix} 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & \cdots & 0 \end{matrix}\right\} M \\ \left.\begin{matrix} 0 & 1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 & \cdots & 0 \end{matrix}\right\} M \\ \begin{matrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{matrix} \\ \left.\begin{matrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \cdots & \vdots & \cdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{matrix}\right\} M \end{bmatrix}_{M^2\times(2M-1)} \tag{6}$$

According to (5), the transmit-receive steering matrix can be expressed as

$$\mathbf{B}(\theta) = \mathbf{G}\mathbf{F}(\theta) \tag{7}$$

Where $\mathbf{F}(\theta)=[\mathbf{f}(\theta_1),\cdots,\mathbf{f}(\theta_K)]$. Then we define $\mathbf{W}=\mathbf{G}^H\mathbf{G}=diag(1,2,\cdots,M-1,M,M-1,\cdots,2,1)$ as a $(2M-1)\times(2M-1)$ diagonal matrix.

Using the reduced-dimension transformation $\mathbf{W}^{-\frac{1}{2}}\mathbf{G}^H$ for the receive signal $\mathbf{Y}(p)$, then we obtain

$$\mathbf{Y}(p)=\mathbf{W}^{-\frac{1}{2}}\mathbf{G}^H\mathbf{Y}(p)=\mathbf{W}^{\frac{1}{2}}\mathbf{F}(\theta)\mathbf{H}(p)+\bar{\mathbf{N}}(p), p=1,2,\ldots,P \quad (8)$$

Where $\bar{\mathbf{N}}(p)=\mathbf{W}^{-\frac{1}{2}}\mathbf{G}^H\mathbf{N}(p)$ is a zero-mean Gaussian white noise vector with covariance matrix $\sigma^2\mathbf{I}_{2M-1}$, $\mathbf{W}^{\frac{1}{2}}\mathbf{F}(\theta)$ can be regarded as the new direction matrix which has lower dimension.

B. Compressed Sensing-Based Noncircular Source for Angle Estimation

According to the reduced-dimension received data in (8) and the properties of noncircular signals, a new received data of virtual array is defined as

$$\mathbf{Z}(p)=\begin{bmatrix}\mathbf{\Pi}_{2M-1}\bar{\mathbf{Y}}^*(p)\\ \bar{\mathbf{Y}}(p)\end{bmatrix}=\begin{bmatrix}\mathbf{\Pi}_{2M-1}\mathbf{W}^{\frac{1}{2}}\mathbf{F}^*(\theta)\mathbf{H}^*(p)\\ \mathbf{W}^{\frac{1}{2}}\mathbf{F}(\theta)\mathbf{H}(p)\end{bmatrix}+\begin{bmatrix}\mathbf{\Pi}_{2M-1}\bar{\mathbf{N}}^*(p)\\ \bar{\mathbf{N}}(p)\end{bmatrix}$$

$$=\bar{\mathbf{G}}\tilde{\mathbf{F}}(\theta)\mathbf{H}(\mathrm{p})+\tilde{\mathbf{N}}(\mathrm{p}), p=1,2,\ldots,P \quad (9)$$

Where
$\bar{\mathbf{G}}=\begin{bmatrix}\mathbf{\Pi}_{2M-1}\mathbf{W}^{\frac{1}{2}}\mathbf{\Pi}_{2M-1} & \mathbf{0}_{(2M-1)}\\ \mathbf{0}_{(2M-1)} & \mathbf{W}^{\frac{1}{2}}\end{bmatrix}$, $\tilde{\mathbf{F}}(\theta)=\begin{bmatrix}\mathbf{\Pi}_{2M-1}\mathbf{F}^*(\theta)\\ \mathbf{F}^*(\theta)\end{bmatrix}$, $\tilde{\mathbf{N}}(\mathrm{p})=\begin{bmatrix}\mathbf{\Pi}_{2M-1}\bar{\mathbf{N}}^*(p)\\ \bar{\mathbf{N}}^*(p)\end{bmatrix}$, $\mathbf{\Pi}_{2M-1}\in\mathbb{C}^{(2M-1)\times(2M-1)}$ is the exchange matrix with ones on its anti-diagonal and zeros elsewhere.

Assumed that the signal components and noise components are independent of each other, so we can get the covariance matrix $\mathbf{R}$ of the reflected signal

$$\mathbf{R}=\frac{1}{P}\sum_{p=1}^{P}\mathbf{Z}(p)\mathbf{Z}^H(p) \quad (10)$$

The singular value decomposition (SVD) of the covariance matrix $\mathbf{R}$ can be written as

$$\mathbf{R}=\begin{bmatrix}\mathbf{E}_s & \mathbf{E}_n\end{bmatrix}\begin{bmatrix}\mathbf{\Sigma} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\mathbf{V}^H \quad (11)$$

Where $\mathbf{E}_s\in\mathbb{C}^{(4M-2)\times K}$ is a signal subspace matrix which is composed of left singular vectors corresponding to the nonzero singular values, $\mathbf{E}_n\in\mathbb{C}^{(4M-2)\times(4M-2-K)}$ is a noise subspace matrix which is composed of left singular vectors corresponding to the zero singular values, $\mathbf{\Sigma}\in\mathbb{C}^{K\times K}$ is a diagonal matrix with the nonzero singular values, $\mathbf{V}$ is a matrix which is composed of right singular vectors corresponding to all the singular value. Then the relationship between signal subspace and the steering vector matrix can be expressed as

$$\mathbf{E}_s=\bar{\mathbf{G}}\tilde{\mathbf{F}}(\theta)\mathbf{T} \quad (12)$$

Where $\mathbf{T}$ is a $K\times K$ nonsingular matrix. Because of $\bar{\mathbf{G}}$ is a $(2M-2)\times(2M-2)$ diagonal matrix, the modified signal subspace can be derived as

$$\tilde{\mathbf{E}}_s=(\bar{\mathbf{G}})^{-1}\mathbf{E}_s=\tilde{\mathbf{F}}(\theta)\mathbf{T} \quad (13)$$

Let $\tilde{\theta}_1,\tilde{\theta}_2,\ldots,\tilde{\theta}_L$ be a sampling grid of all target locations of interest. We construct a matrix composed of steering vectors corresponding to each potential target location as its columns, due to the fact that received data matrix is a conjugate matrix, the new complete dictionary can be written as

$$\mathbf{\Theta}=\begin{bmatrix}\mathbf{\Pi}_{2M-1}\mathbf{H}^*\\ \mathbf{H}\end{bmatrix} \quad (14)$$

Then construct a matrix $\mathbf{\Psi}\in\mathbb{C}^{L\times K}$, which the rows of $\mathbf{\Psi}$ corresponding to the true DOAs and with the other rows being all-zero. And (13) can also be expressed as

$$\tilde{\mathbf{E}}_s=\mathbf{\Theta}\mathbf{\Psi} \quad (15)$$

Stacking each succeeding column of $\tilde{\mathbf{E}}_s$, $\mathbf{r}=vec(\tilde{\mathbf{E}}_s)$, then (15) can be described as

$$\mathbf{r}=(\mathbf{I}_K\otimes\mathbf{\Theta})vec(\mathbf{\Psi})=\mathbf{\Phi}\mathbf{g} \quad (16)$$

Where $\mathbf{\Phi}=\mathbf{I}_K\otimes\mathbf{\Theta}\in\mathbb{C}^{K(4M-2)\times LK}$, $\mathbf{g}=vec(\mathbf{\Psi})\in\mathbb{C}^{LK\times 1}$ is a sparse vector. Then (15) can be translated into the sparse recovery problem in compressed sensing

$$\min\|\mathbf{g}\|_0, s.t.\ \ \mathbf{r}=\mathbf{\Phi}\mathbf{g} \quad (17)$$

Where $\|\mathbf{g}\|_0$ denotes the zero-norm of $\mathbf{g}$, and the sparse vector $\mathbf{g}$ can be obtained by using OMP method [18]. After recovering the matrix $\mathbf{g}$, the matrix $\boldsymbol{\Psi}$ can be estimated by reverse transformation, which the nonzero rows or the relatively large rows in $\boldsymbol{\Psi}$ will show the DOAs of the targets. The detailed recovery processing via OMP is shown in Fig.2.

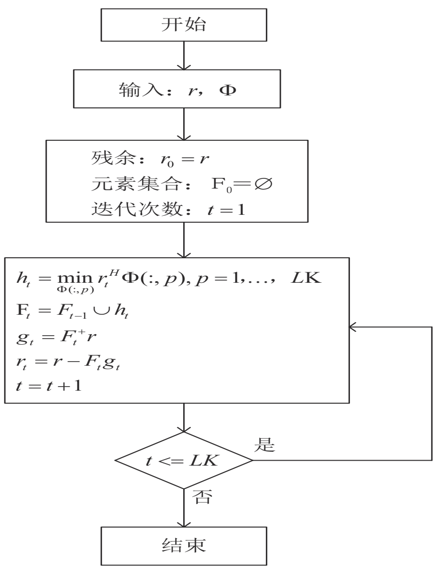

Fig.2. The OMP algorithm flow

Up to now, we have achieved the compressed sensing-based angle estimation for noncircular source in MIMO radar, and the procedure of our algorithm is summarized as following:

（1） Matched filter using $\mathbf{S}^H$ to obtain the received data vector $\mathbf{Y}(p) \in \mathbb{C}^{M^2 \times 1}$, and obtain the lower-dimensional data vector $\bar{\mathbf{Y}}(p) \in \mathbb{C}^{(2M-1)\times 1}$ by using the reduced dimension matrix.

（2） Utilize the properties of noncircular signal to construct an extended matrix $\mathbf{Z}(p) \in \mathbb{C}^{(4M-2)\times 1}$ as showing in (9).

（3） Compute the covariance matrix $\mathbf{R}$ and get the signal subspace $\mathbf{E}_s$ by SVD of $\mathbf{E}_s$; obtain the modified signal subspace $\tilde{\mathbf{E}}_s$ using (13).

（4） Stack each succeeding column of $\tilde{\mathbf{E}}_s$, $\mathbf{r} = vec(\tilde{\mathbf{E}}_s)$, construct the new complete dictionary and the recovery constraint equation of compressed sensing.

（5） Use OMP method to recovery the sparse vector $\mathbf{g}$, and estimate the true DOAs of target by the nonzero rows or the relatively large rows in $\boldsymbol{\Psi}$.

IV. Simulation Results

In this section, several numerical examples are presented to verify the effectiveness of the proposed method. We define the root mean squared error (RMSE) as

$$RMSE = \frac{1}{K}\sum_{k=1}^{K}\sqrt{\frac{1}{L}\sum_{l=1}^{L}[(\hat{\theta}_{k,l}-\theta_k)^2]} \qquad (18)$$

Where $\hat{\theta}_{k,l}$ is the estimation of DOA of θ_k the lth Monte Carlo trial, L is the number of Monte trials.

Consider a monostatic MIMO radar system with $M = N = 10$ elements used for both transmit and receive arrays. Both the transmit and receive arrays are ULA, and the inter-element spaces of the transmit and receive arrays are half-wavelength. The number of snapshots is $J = 50$ .We assume there are $K = 3$ targets with angle $(\theta_1, \theta_2, \theta_3)=(-10^\circ, 5^\circ, 15^\circ)$ .The DOA estimation result of the targets with $SNR = 20dB$ is showing in Fig.1.

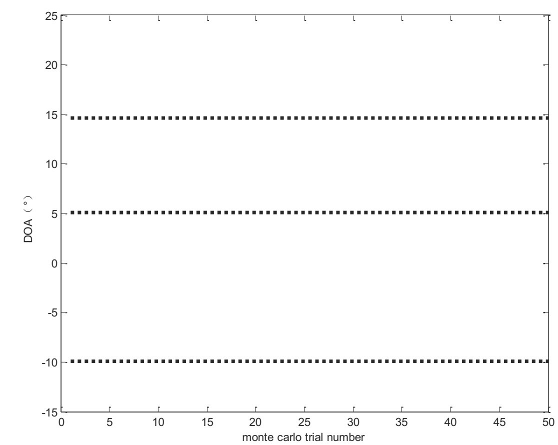

Fig 1. Angle estimation result of the proposed algorithm

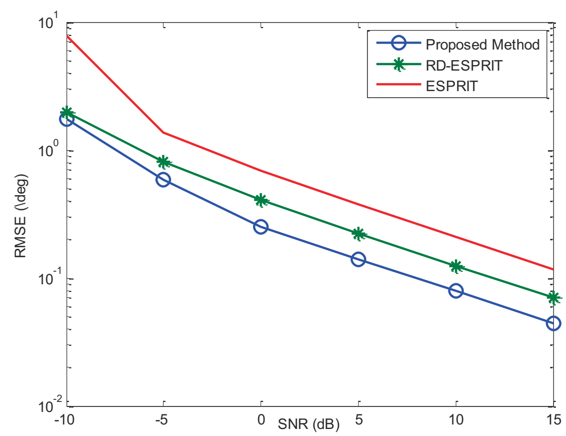

Fig 2. Angle estimation performance comparison

We compare the proposed algorithm against the RD-ESPRIT algorithm and ESPRIT algorithm.Fig.2 presents the comparison for the algorithms. From Fig.2, we can know that the angle estimation performance of the proposed algorithm is better than that of RD-ESPRIT algorithm and ESPRIT algorithm.

From Fig.3, we can find that the performance of angle estimation of the proposed algorithm and the diversity gain increase with the number of array elements increasing.

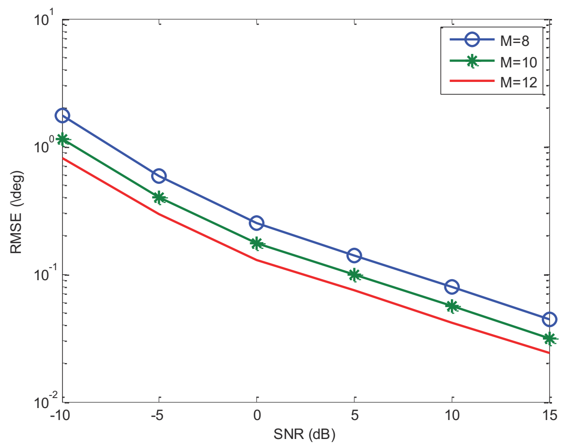

Fig 3. RMSE versus SNR for different sensors

V. CONCLUSION

In this paper, we propose an angle estimation algorithm based on the extended matrix compressed sensing. According to the properties of noncircular signals, constructing an extended matrix, and then estimating the true DOAs of target using OMP method. Therefore, the angle estimation performance of the proposed algorithm is better than that of RD-ESPRIT algorithm and ESPRIT algorithm. Furthermore, the proposed algorithm requires no knowledge of the noise. Several simulation results have verified the effectiveness of the proposed algorithm.

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