PAPER

Real-Valued Reweighted l_1 Norm Minimization Method Based on Data Reconstruction in MIMO Radar*

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SUMMARY In this paper, a real-valued reweighted l_1 norm minimization method based on data reconstruction in monostatic multiple-input multiple-output (MIMO) radar is proposed. Exploiting the special structure of the received data, and through the received data reconstruction approach and unitary transformation technique, a one-dimensional real-valued received data matrix can be obtained for recovering the sparse signal. Then a weight matrix based on real-valued MUSIC spectrum is designed for reweighting l_1 norm minimization to enhance the sparsity of solution. Finally, the DOA can be estimated by finding the non-zero rows in the recovered matrix. Compared with traditional l_1 norm-based minimization methods, the proposed method provides better angle estimation performance. Simulation results are presented to verify the effectiveness and advantage of the proposed method.

key words: *MIMO radar, DOA estimation, sparse representation, real-valued reweighted l_1 norm minimization*

1. Introduction

Direction of arrival (DOA) estimation of far-field narrowband signal has drawn considerable attention in the past few decades [1]–[3]. Multiple-input multiple-output (MIMO) radar uses multiple antennas to simultaneously transmit diverse waveforms instead of coherent waveforms and utilizes multiple antennas to receive the reflected signals. It has been verified that MIMO radar has a lot of potential advantages over the conventional phased-array radar [4].

Angle estimation of multiple targets from received signals corrupted by noise is an important attribute of array signal processing and MIMO radar. Towards this goal, many subspace-based algorithms have been developed for MIMO radar such as multiple signal classification (MUSIC) algorithm [5], reduced-complexity Capon (RC-Capon) algorithm [6], estimation of signal parameters via rotational techniques (ESPRIT) algorithm [7] and so on. In [8], RD-ESPRIT is proposed for DOA estimation in monistatic MIMO radar by exploiting the special configuration of the virtual array, and the angle estimation performance is improved. Additionally, a conjugate unitary ESPRIT (CU-ESPRIT) [9] is proposed for angle estimation, which exploits the characteristic of the non-circular signals to enlarge the aperture of monostatic MIMO radar and provides better angle estimation performance. However, most of algorithms mentioned above are not suitable to limited snapshots, low SNR and closely spaced sources.

Recently, the emerging field of sparse signal representation (SSR), has attracted significant interest for DOA estimation [10]–[14]. In [10], Dmitry M. et al. cast the DOA estimation problem into a sparse signal recovery problem and proposed l_1-SVD approach to enforce the sparsity of the solution, in which the singular value decomposition (SVD) technique is used to simplify calculation. However, it has an important drawback that larger coefficients are penalized more heavily than smaller coefficients. In contrast, J. Yin et al. use the sparse representation of the array covariance vectors (SRACV) for DOA estimation in [11]. Thus, a new DOA estimation method using a covariance-based sparse representation in the presence of unknown nonuniform noise is proposed in [12]. Additionally, in [13], a SRSSV-l_0 method for DOA and power estimation is proposed by Y. Tian et al., using a sparse representation of second-order statistics vector and l_0 norm approximation. The DOA estimation performance is reasonably improved in these covariance sparse representation-based algorithms. However, the computational complexity grows proportionally with the number of sources or sensors. Furthermore, a real-valued l_1-SVD (RV l_1-SVD) is proposed in [14], which provides slight better angle estimation performance than l_1-SVD algorithm and lower computational complexity than these covariance sparse representation-based algorithms. Compared with traditional subspace-based methods, these sparse representation-based methods can be better adaptable to many scenarios, such as limited snapshots, low SNR and coherent targets. However, according to the l_1 norm penalty minimization, it can be concluded that the DOA estimation performance of sparse representation-based methods will be mainly limited by two drawbacks: 1) two-dimensional overcomplete dictionary and multiple measurement vector (MMV) problem are involved in the sparse signal reconstruction process, in which the high computational burden perhaps makes the recovery method fail. 2) larger coefficients are penalized more heavily than smaller coefficients and the sparsest solution of l_1 norm penalty does not approximate to the l_0 norm penalty.

In this paper, we propose a real-valued reweighted l_1 norm minimization method based on data reconstruction in

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monostatic multiple-input multiple-output (MIMO) radar. Exploiting the special structure of the received data, a data reconstruction approach and unitary transformation technique are used to transform the received data into a one-dimensional real-valued one. Then a weight matrix based on real-valued MUSIC spectrum is designed for reweighting l_1 norm minimization to enhance the sparsity of solution. The proposed method involves the data reconstruction approach and a real-valued process, which implies lower computational complexity than complex-valued multiple measurement vector (MMV) problem. Thus, the proposed method provides better angle estimation performance than l_1-SVD and RV l_1-SVD algorithms.

The reminder of this paper is organized as follows. In Sect. 2, we briefly depict the problem formulation. In Sect. 3, we have shown the proposed method of DOA estimation as a sparse representation scheme involving data reconstruction approach, real-valued process and reweighting l_1 norm minimization. The performance analysis of the proposed method is evaluated in Sect. 4. Several simulation results verify the performance of the proposed method in Sect. 5. Finally, Sect. 6 concludes this paper.

Notation: $(\cdot)^H$, $(\cdot)^T$, $(\cdot)^*$ represent the Hermitian transpose, transpose, complex conjugate, respectively. $\otimes$ denotes the Kronecker operator, diag$(\cdot)$ denotes the diagonalization operation. vec$(\cdot)$ denotes a matrix operation that stacks the columns of a matrix under each other to form a new vector, I_M and 0_M are a $M\times M$ dimensional unit matrix and a $M\times M$ diagonal matrix with all elements equal 0, respectively. Furthermore, $\|\cdot\|_1$ and $\|\cdot\|_F$ represent the l_1 norm and Frobenius norm, respectively.

2. Problem Formulation

Consider a narrow-band monostatic MIMO radar system equipped with M transmit antennas and N receive antennas. The inter-element spaces of the transmit and receive arrays are half-wavelength, M transmit antennas emit M different orthogonal narrowband waveforms, which have identical bandwidth and centre frequency. After matched filtering, the output at the receive array can be expressed as [14]

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{w}(t) \tag{1}$$

where $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T$ is the $K \times 1$ zero-mean signal vector, $\mathbf{w}(t)$ is the $MN \times 1$ Gaussian white noise vector with zero mean and covariance matrix $\sigma^2\mathbf{I}_{MN}$, $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_K]$ is an $MN \times K$ array manifold matrix, depending on the unknown DOAs, so it is not known. $\mathbf{a}_k = \mathbf{a}_t(\theta_k)\otimes\mathbf{a}_r(\theta_k)$ is a steering vector of the *kth* target. The transmit steering vector $\mathbf{a}_t(\theta_k)$ and the receive steering vector $\mathbf{a}_r(\theta_k)$ are showed as:

$$\mathbf{a}_t(\theta_k) = \begin{cases} [e^{-j\frac{M-1}{2}\pi\sin\theta_k}, \ldots, e^{-j\frac{1}{2}\pi\sin\theta_k}, \\ \qquad e^{j\frac{1}{2}\pi\sin\theta_k}, \ldots, e^{j\frac{M-1}{2}\pi\sin\theta_k}]^T, & M \text{ is even} \\ [e^{-j\frac{M-1}{2}\pi\sin\theta_k}, \ldots, 1, \ldots, e^{j\frac{M-1}{2}\pi\sin\theta_k}]^T, & M \text{ is odd} \end{cases}$$

and

$$\mathbf{a}_r(\theta_k) = \begin{cases} [e^{-j\frac{N-1}{2}\pi\sin\theta_k}, \ldots, e^{-j\frac{1}{2}\pi\sin\theta_k}, \\ \qquad e^{j\frac{1}{2}\pi\sin\theta_k}, \ldots, e^{j\frac{N-1}{2}\pi\sin\theta_k}]^T, & N \text{ is even} \\ [e^{-j\frac{N-1}{2}\pi\sin\theta_k}, \ldots, 1, \ldots, e^{j\frac{N-1}{2}\pi\sin\theta_k}]^T, & N \text{ is odd} \end{cases}$$

By collecting L snapshots, the received data matrix can be rewritten as:

$$\begin{aligned} \mathbf{X} &= [\mathbf{x}(1), \mathbf{x}(2), \ldots, \mathbf{x}(L)] = \mathbf{AS} + \mathbf{W} \\ &= \begin{bmatrix} \mathbf{A}_r\mathbf{D}_1(\mathbf{A}_t)\mathbf{S} \\ \mathbf{A}_r\mathbf{D}_2(\mathbf{A}_t)\mathbf{S} \\ \vdots \\ \mathbf{A}_r\mathbf{D}_N(\mathbf{A}_t)\mathbf{S} \end{bmatrix} + \begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \\ \vdots \\ \mathbf{W}_N \end{bmatrix} \end{aligned} \tag{2}$$

where $\mathbf{A}_t = [\mathbf{a}_t(\theta_1), \mathbf{a}_t(\theta_2), \ldots, \mathbf{a}_t(\theta_k)]$ is the transmit steering matrix composed of transmit steering vectors and $\mathbf{A}_r = [\mathbf{a}_r(\theta_1), \mathbf{a}_r(\theta_2), \ldots, \mathbf{a}_r(\theta_k)]$ is the receive steering matrix composed of receive steering vectors. $\mathbf{X} = [\mathbf{x}(t_1), \mathbf{x}(t_2), \ldots, \mathbf{x}(t_L)]$ is a received data matrix, $\mathbf{S} = [\mathbf{s}(t_1), \mathbf{s}(t_2), \ldots, \mathbf{s}(t_L)]$ is signal matrix and $\mathbf{W} = [\mathbf{w}(t_1), \mathbf{w}(t_2), \ldots, \mathbf{w}(t_L)]$ is sensor noise matrix. $\mathbf{D}_n(\cdot)$ is a diagonal matrix with *nth* row of the matrix.

Remark 1: the two-dimensional dictionary is very large, for example, if there are 5 potential DOAs and $M = N = 10$, the transmit-receive dictionary matrix will have 100 rows and 25 columns, which will result in high computational burden, and perhaps make the recovery methods fail. The traditional methods [6], [15], [16] reduce the computational complexity by constructing a reduced-dimensional transformation matrix, based on the structure of the transmit-receive steering vector. In the next section, a novel reduced-dimensional method is presented to reduce complexity, based on the received data reconstruction approach.

3. DOA Estimation Method-Based Data Reconstruction Utilizing Real-Valued Reweighted l_1 Norm Minimization in MIMO Radar

If the received data matrix is directly used for sparse representation scheme, it leads very high computational complexity or even perhaps makes the sparse signal recovery algorithm fail. Therefore, we introduce a received data reconstruction approach to reduce the computational complexity in the following. Defining $\mathbf{X}_m \in \mathbf{C}^{N\times L}$ denotes the $[(m-1)N+1]th$ row to $(mN)th$ row of $\mathbf{X}$, we can get $\mathbf{X}_m = \mathbf{A}_r\mathbf{D}_n(\mathbf{A}_t)\mathbf{S} + \mathbf{W}_m, m = 1, 2, \ldots, M$. According to the structure of the transmit-receive steering vector, rearranging the received data, then we can reconstruct a novel received data matrix as follows:

$$\begin{aligned} \bar{\mathbf{X}} &= [\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_M] \\ &= [\mathbf{A}_r\mathbf{D}_1(\mathbf{A}_t)\mathbf{S} + \mathbf{W}_1, \mathbf{A}_r\mathbf{D}_2(\mathbf{A}_t)\mathbf{S} + \mathbf{W}_2, \ldots, \\ &\qquad \mathbf{A}_r\mathbf{D}_N(\mathbf{A}_t)\mathbf{S} + \mathbf{W}_M] \\ &= \mathbf{A}_r[\mathbf{D}_1(\mathbf{A}_t)\mathbf{S}, \mathbf{D}_2(\mathbf{A}_t)\mathbf{S}, \ldots, \mathbf{D}_N(\mathbf{A}_t)\mathbf{S}] \\ &\qquad + [\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_M] \\ &= \mathbf{A}_r(\mathbf{S}\odot\mathbf{A}_t)^T + [\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_M] \\ &= \mathbf{B}\bar{\mathbf{S}} + \bar{\mathbf{W}} \end{aligned} \tag{3}$$

where $\bar{\mathbf{S}} = (\mathbf{S}\odot\mathbf{A}_t)^T \in \mathbf{C}^{K\times ML}$. From Eq. (3), it can be shown that the two-dimensional angles (i.e. DODs and DOAs) are separated, and $\mathbf{B} = \mathbf{A}_r$ which contains only one-dimensional angle information (i.e. DOAs) can be considered as the new one-dimensional dictionary matrix with respect to Eq. (2). Then the DOAs can be estimated based on the sparse representation scheme. And $\bar{\mathbf{W}} = [\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_M] \in \mathbf{C}^{N\times ML}$ is the new noise matrix after the reconstruction.

Remark 2: Based on the received data reconstruction approach, we are capable of constructing a new received data matrix with one-dimensional dictionary, and the sample size virtually transforms from L to ML. Additionally, if the SVD technique is directly used for calculating the equation (3), the received signal matrix multiplying the right singular matrix (denoted as a sparse received matrix) has lower dimension than the sparse received matrix in the traditional method [17] using reduced-dimensional transformation matrix, which can reduce the computational complexity. For example, if the received data matrix has MN rows and L columns, under the framework of the sparse representation, the sparse received matrix is a $N \times K$ dimensional matrix, while in traditional methods is $(N + M - 1) \times K$.

Next, an augmented sample matrix is considered as $\mathbf{Y} = [\ \bar{\mathbf{X}} \quad \mathbf{\Gamma}_N \bar{\mathbf{X}}^* \mathbf{\Gamma}_{ML}\ ]$, where $\mathbf{\Gamma}_M$ denotes a $N{\times}N$ exchange matrix with ones on its anti-diagonal and zeros elsewhere. It is shown that $\mathbf{Y}$ is centro-Hermitian, and can be transformed to a real-valued matrix through the unitary transformation according to [18], [19]

$$\mathbf{Y}_r = \mathbf{U}_N^H [\ \bar{\mathbf{X}} \quad \mathbf{\Gamma}_N \bar{\mathbf{X}}^* \mathbf{\Gamma}_{ML}\ ] \mathbf{U}_{2ML} \tag{4}$$

where $\mathbf{U}$ is a unitary transformation matrix, defined as:

For N even,

$$\mathbf{U}_{2K} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_K & j\mathbf{I}_K \\ \mathbf{\Gamma}_K & -j\mathbf{\Gamma}_K \end{bmatrix} \tag{5}$$

For N odd,

$$\mathbf{U}_{2K+1} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_K & 0 & j\mathbf{I}_K \\ 0 & \sqrt{2} & 0 \\ \mathbf{\Gamma}_K & 0 & -j\mathbf{\Gamma}_K \end{bmatrix} \tag{6}$$

and it can be concluded from Eq. (4) that the sample size is virtually doubled from ML to $2ML$ due to incorporate forward-backward averaging. On the other hand, the over-complete dictionary $\hat{\mathbf{B}}$ satisfies:

$$\mathbf{B\Phi} = \mathbf{\Gamma}_N (\mathbf{B\Phi})^* \tag{7}$$

where $\mathbf{\Phi} = \mathrm{diag}[e^{-j(N-1)/2\sin\theta_1}, \ldots, e^{-j(N-1)/2\sin\theta_K}]$, and the real-valued steering matrix can be expressed as $\mathbf{B}_r = \mathbf{U}_N^H \mathbf{B\Phi}$ after unitary transformation, which can also be denoted as

$$\mathbf{B}_r = \begin{cases} \sqrt{2}[\cos(-j\frac{N-1}{2}\pi\sin\theta_k), \ldots, \cos(-j\frac{1}{2}\pi\sin\theta_k), \\ \sin(j\frac{1}{2}\pi\sin\theta_k), \ldots, \pi\sin(j\frac{N-1}{2}\sin\theta_k)]^T, \quad N \text{ is even} \\ \sqrt{2}[\cos(-j\frac{N-1}{2}\pi\sin\theta_k), \ldots, \\ \qquad 1, \ldots, \pi\sin(j\frac{N-1}{2}\sin\theta_k)]^T, \quad N \text{ is odd} \end{cases}$$

Thus, after some straightforward algebraic manipulations, the Eq. (4) can be rewritten as:

$$\begin{aligned} \mathbf{Y}_r &= \mathbf{U}_N^H [\ \mathbf{B}\bar{\mathbf{S}} \quad \Gamma_N \mathbf{B}^* \bar{\mathbf{S}}^* \Gamma_{ML}\ ] \mathbf{U}_{2ML} \\ &\qquad + \mathbf{U}_N^H [\ \bar{\mathbf{W}} \quad \Gamma_N \bar{\mathbf{W}}^* \ \Gamma_{ML}] \mathbf{U}_{2ML} \\ &= \mathbf{B}_r \bar{\mathbf{S}}_r + \bar{\mathbf{W}}_r \end{aligned} \tag{8}$$

where $\bar{\mathbf{S}}_r = [\ \mathbf{\Phi}^* \bar{\mathbf{S}} \quad \mathbf{\Phi} \bar{\mathbf{S}}^* \Gamma_{ML}\ ] \mathbf{U}_{2ML}$ is the real-valued signal matrix, and $\bar{\mathbf{W}}_r = \mathbf{U}_N^H [\ \bar{\mathbf{W}} \quad \Gamma_N \bar{\mathbf{W}}^* \Gamma_{ML}\ ] \mathbf{U}_{2ML}$ is the real-valued noise matrix. According to the above analysis, it can be found that the unitary transformation can further reduce the computational complexity. Additionally, to mitigate the effect of the additive noise, the singular value decomposition (SVD) technique [10] of $\mathbf{Y}_r$ is introduced.

$$\mathbf{Y}_r = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{V}_s^H + \mathbf{U}_n \mathbf{\Lambda}_n \mathbf{V}_n^H \tag{9}$$

where $\mathbf{U}_s$ and $\mathbf{V}_s$ are composed with the real-valued singular vectors corresponding to the K largest singular values. $\mathbf{U}_n$ and $\mathbf{V}_n$ are composed with the real-valued singular vectors corresponding to the residual singular values. $\mathbf{\Lambda}_s$ and $\mathbf{\Lambda}_n$ are diagonal matrices whose diagonal elements are the K largest singular values and the residual singular values, respectively. Then we can get

$$\mathbf{Y}_{rsv} = \mathbf{B}_r \bar{\mathbf{S}}_r \mathbf{V}_s + \bar{\mathbf{W}}_r \mathbf{V}_s = \mathbf{B}_r \bar{\mathbf{S}}_{rsv} + \bar{\mathbf{W}}_{rsv} \tag{10}$$

where $\bar{\mathbf{S}}_{rsv} = \bar{\mathbf{S}}_r \mathbf{V}_s$ and $\bar{\mathbf{W}}_{rsv} = \bar{\mathbf{W}}_r \mathbf{V}_s$.

Let $\{\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_J\}$ be the discretized sampling grid of the DOAs of interest, and $\{\hat{\theta}_i\}_{i=1}^J (J \gg N > K)$ denotes a grid that covers the possible locations. Then the one-dimensional real-valued complete dictionary can be expressed as $\bar{\mathbf{B}}_r^{\hat{\theta}} = \mathbf{U}_N^H \mathbf{B}_r^{\hat{\theta}} \Phi^{\hat{\theta}}$ with $\mathbf{B}_r^{\hat{\theta}} = [\mathbf{b}(\hat{\theta}_1), \mathbf{b}(\hat{\theta}_2), \ldots, \mathbf{b}(\hat{\theta}_J)]$ and $\Phi^{\hat{\theta}} = \mathrm{diag}[e^{-j(N-1)/2\sin\hat{\theta}_1}, e^{-j(N-1)/2\sin\hat{\theta}_2}, \ldots, e^{-j(N-1)/2\sin\hat{\theta}_J}]$, where $\mathbf{b}(\hat{\theta}_j)$ is a vector that its elements are composed of steering vectors corresponding to the potential DOAs. Based on the sparsity of targets corresponding to the whole spatial space, the signal model in Eq. (10) can be converted into a sparse representation model.

$$\bar{\mathbf{Y}}_{rsv} = \bar{\mathbf{B}}_r^{\hat{\theta}} \bar{\mathbf{S}}_{rsv}^{\hat{\theta}} + \bar{\mathbf{W}}_{rsv}^{\hat{\theta}} \tag{11}$$

where $\bar{\mathbf{Y}}_{rsv}$ is a sparse received data matrix. $\bar{\mathbf{S}}_{rsv}^{\hat{\theta}}$ and $\bar{\mathbf{S}}_r$ have the same rows support, which means $\bar{\mathbf{S}}_{rsv}^{\hat{\theta}}$ only has a few non-zero rows corresponds to DOAs of the possible targets, i.e., the matrix $\bar{\mathbf{S}}_{rsv}^{\hat{\theta}}$ is sparse. Thus, the sparse signal $\bar{\mathbf{S}}_{rsv}^{\hat{\theta}}$ can be estimated by converting the Eq. (11) into a l_1 norm minimization problem, and the DOAs can be estimated by finding the positions of non-zero rows of $\bar{\mathbf{S}}_{rsv}^{\hat{\theta}}$. However, the sparsest solution of l_1 norm penalty does not approximate to the l_0 norm penalty effectively, the sparse signal reconstruction accuracy is limited.

After the above processing involving data reconstruction and real-valued transformation, a real-valued reweighted l_1 norm minimization method based on data reconstruction for DOA estimation in monostatic MIMO radar is proposed by exploiting the orthogonality of the real-valued steering vector and its corresponding noise subspace. The real-valued dictionary matrix $\bar{\mathbf{B}}_r^{\hat{\theta}}$ can be divided into two parts, which is expressed as $\bar{\mathbf{B}}_r^{\hat{\theta}} = [\ \bar{\mathbf{B}}_{r1}^{\hat{\theta}} \quad \bar{\mathbf{B}}_{r2}^{\hat{\theta}}\ ]$, and $\bar{\mathbf{B}}_{r1}^{\hat{\theta}}$ is composed with the real-valued steering vector $\mathbf{U}_N^H \mathbf{B}_{r1}^{\hat{\theta}} \Phi_1^{\hat{\theta}}$

corresponding to the potential targets, and $\bar{\mathbf{B}}_{r2}^{\hat{\theta}} = \mathbf{U}_N^H \mathbf{B}_{r2}^{\hat{\theta}} \Phi_2^{\hat{\theta}}$ is composed with the residual real-valued steering vector of $\bar{\mathbf{B}}_r^{\hat{\theta}}$. Then we have the following equation:

$$(\bar{\mathbf{B}}_r^{\hat{\theta}})^H \mathbf{V}_n \mathbf{V}_n^H \bar{\mathbf{B}}_r^{\hat{\theta}} = \begin{bmatrix} (\bar{\mathbf{B}}_{r1}^{\hat{\theta}})^H \mathbf{V}_n \mathbf{V}_n^H \bar{\mathbf{B}}_r^{\hat{\theta}} \\ (\bar{\mathbf{B}}_{r2}^{\hat{\theta}})^H \mathbf{V}_n \mathbf{V}_n^H \bar{\mathbf{B}}_r^{\hat{\theta}} \end{bmatrix} = \begin{bmatrix} \mathbf{W}_1^T & \mathbf{W}_2^T \end{bmatrix} \tag{12}$$

The weighted matrix can be designed as follows:

$$\hat{\mathbf{W}} = \mathrm{diag}([(\mathbf{W}_1^{(l_2)})^T, (\mathbf{W}_2^{(l_2)})^T])/\max(\mathbf{W}_1^{(l_2)}) \tag{13}$$

It can be concluded that when the snapshots $T \to \infty$, the weight vectors $\mathbf{W}_{1,j}^{(l_2)} \to 0$ and $\mathbf{W}_{2,j}^{(l_2)} > 0$, where $\mathbf{W}_{1,j}^{(l_2)}$ and $\mathbf{W}_{2,j}^{(l_2)}$ denote the *jth* elements of the $\mathbf{W}_1^{(l_2)}$ and $\mathbf{W}_2^{(l_2)}$, respectively. Consequently, the weighted matrix $\hat{\mathbf{W}}$ is employed to achieve the idea that those entries who approximated to the non-zero in recovered matrix are penalized by larger weights and the other entries are punished by small weights. Finaly, the real-valued reweighted l_1 norm minimization method based on the data reconstruction for DOA estimation is formulated as:

$$\min \left\| \hat{\mathbf{W}} (\bar{\mathbf{S}}_{rsv}^{\hat{\theta}})^{l_2} \right\|_1 \text{ s.t. } \left\| \bar{\mathbf{Y}}_{rsv} - \bar{\mathbf{B}}_r^{\hat{\theta}} \bar{\mathbf{S}}_{rsv}^{\hat{\theta}} \right\|_F \le \eta \tag{14}$$

where η is the regularization parameter which sets the amount of error and plays important role in the final DOA estimation performance. A large η emphasizes the role of the l_1-term, which may cause wrong target parameter estimation. A small η emphasizes the role of the l_2-term, which may produce many spurious peaks in spatial spectrum. Usually, the confidence interval for choosing the regularization parameter is enough to set as 99%. Equation (14) can be efficiently calculated by SOC (second order cone) programming software packages such as SeDuMi [20] and CVX [21]. Then the DOAs estimation are obtained by plotting $\bar{\mathbf{S}}_{rsv}^{\hat{\theta}}$, solved from (14).

4. Performance Analysis

Remark 3: Regarding the computational complexity, we consider the major parts: the formulation of the weighted matrix $\hat{\mathbf{W}}$ and the sparse reconstruction process in Eq. (14). The formulation of the weighted matrix $\hat{\mathbf{W}}$ requires $O\{LN^2 + N^3 + JN(N-K)\}$. According to the conclusion in [10], solving Eq. (14) through the SOC programming requires $O(K^3 J^3)$. Due to the real-valued transformation, the computational complexity is decreased by a factor of at least four, which means the computational cost of the sparse reconstruction process in Eq. (14) can be derived as $O(\frac{1}{4}K^3 J^3)$. While for the l_1-SVD and RV l_1-SVD algorithms, the main computational cost is also the sparse reconstruction process, which require $O(K^3 J^3)$ and $O(\frac{1}{4}K^3 J^3)$, respectively. In Table 1, the computational complexity of the proposed method is compared with that of the l_1-SVD and RV l_1-SVD algorithms. Although the proposed method utilizes the data reconstruction method to reduce the complexity of sparsity signal reconstruction, the computational cost of the proposed method is higher than l_1-SVD and RV l_1-SVD algorithms because the weighted matrix is involved in the proposed method. However, the advantages of the proposed method outweigh the cost of additional computation: the proposed method can achieve more accurate DOA estimation and higher capability of resolving closely spaced targets than both of the l_1-SVD and RV l_1-SVD algorithms.

Table 1 Computational complexity.

Estimators	major computational complexities
l_1-SVD	$K^3 J^3$
RV l_1-SVD	$\frac{1}{4}K^3 J^3$
The proposed method	$LN^2 + N^3 + JN(N-K) + (\frac{1}{4}K^3 J^3)$

The proposed method can enhance the DOA estimation performance and has higher angular resolution probability than l_1-SVD and RV l_1-SVD algorithms, the reasons are that the SNR gain for the received data is enhanced and the virtual aperture is enlarged in the proposed method; the proposed method has a better noise suppression through exploiting the real structure of the data matrix and the weight matrix. This advantages are to be verified in simulation results.

5. Simulation Results

In this section, the performance of the proposed method is investigated, and compared with l_1-SVD [10], RV l_1-SVD [14]. A monostatic MIMO radar with $M = N = 6$ is considered in all simulation results. The inter-element spacing is half a wavelength and both of the transmit arrays and receive arrays are ULAs. Besides, 100 Monte Carlo trials are carried for statistical calculation in all methods. The direction grid is uniform with 0.01° sampling from −90° to 90° and the confidence interval for choosing the regularization parameter is set as 99% in all algorithms. In the following simulations, we defined the signal-to-noise ratio (SNR) as $\mathrm{SNR} = 10\log_{10}(\|\mathbf{AW}\|_F^2 / \|\mathbf{W}\|_F^2)$. In the paper, the root mean square error (RMSE) that evaluates the performance of DOA estimation is defined as follows:

$$\mathrm{RMSE} = \sqrt{\frac{1}{100K} \sum_{i=1}^{100} \sum_{k=1}^{K} (\hat{\theta}_{k,i} - \theta_k)^2} \tag{15}$$

where $\hat{\theta}_{k,i}$ is the estimate of DOA of the *kth* signal in the *ith* Monte Carlo trial.

Figure 1 shows the spatial spectra of the proposed method for three uncorrelated targets, where SNR=5 dB and L=200. The three uncorrelated targets are −10°, 5° and 10°. From Fig. 1, it can be seen that the proposed method provides sharp peaks, which means that the proposed method has capable of resolving three uncorrelated targets.

Figure 2 shows the spatial spectra of the l_1-SVD, RV l_1-SVD algorithms and the proposed method for two uncorrelated and closely spaced targets, where SNR=5 dB and L=200. The two uncorrelated targets are fixed at −0.5° and 5.5° with closed spaced 6°. From Fig. 2, it can seen that all

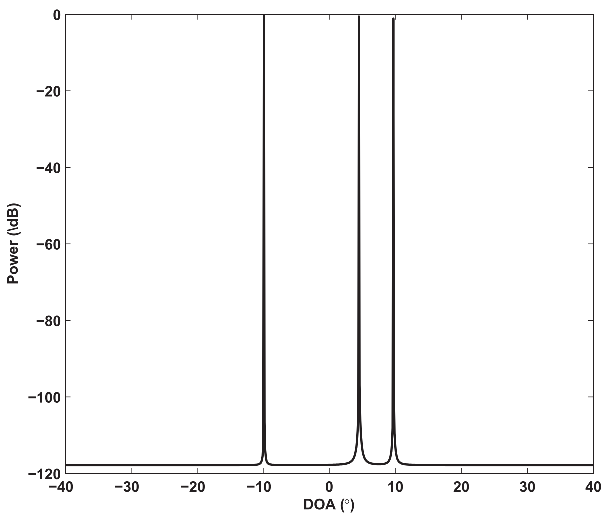

Fig. 1 Spatial spectra of the proposed method for three uncorrelated targets.

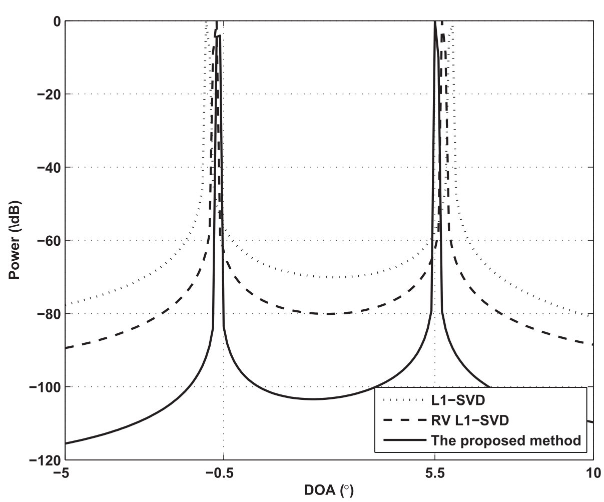

Fig. 2 Spatial spectra of l_1-SVD, RV l_1-SVD and the proposed method for two uncorrelated and closely spaced targets.

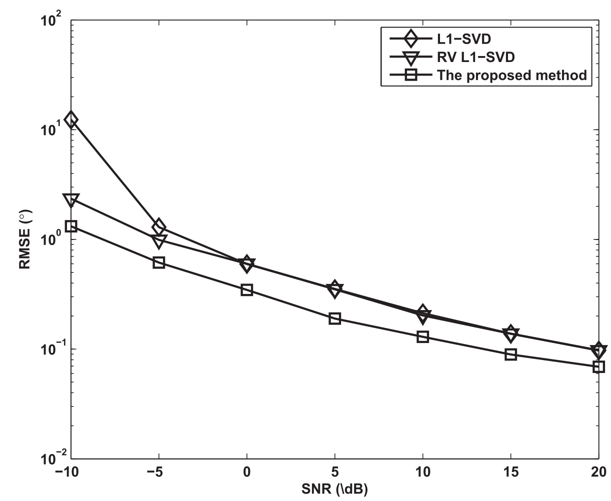

Fig. 3 RMSE of DOA estimation against SNR.

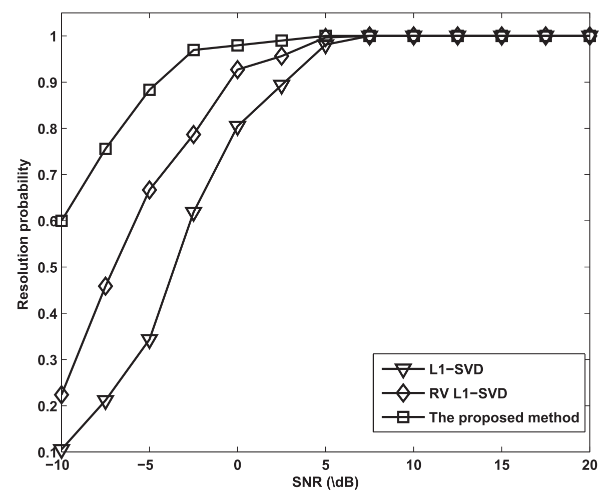

Fig. 4 Resolution probability versus angle separation.

methods can resolve two targets but peaks of the proposed method are more closer to the true DOAs than the l_1-SVD, RV l_1-SVD algorithms, which means the proposed method can achieve more accurate DOA estimation than the l_1-SVD, RV l_1-SVD algorithms.

Figure 3 shows the RMSE of DOA estimation via SNR with different algorithms, where L=200 and two uncorrelated targets are $\theta_1 = -0.5°, \theta_2 = 10.5°$. We vary the SNR from −10 dB to 20 dB in 5 dB step. From Fig. 3, it can be seen that the proposed method performs better angle estimation performance than the l_1-SVD and RV l_1-SVD algorithms (especially in low SNR region) owing to that the SNR gain is enhanced and those entries who are more likely to be zero in recovered matrix are punished by larger weights.

Figure 4 shows the capability of resolving closely spaced targets by three different algorithms versus SNR, where the number of snapshots is fixed at 200 and SNR varies from −10 dB to 20 dB in 2.5 dB step. Two uncorrelated targets are simulated with directions as $\theta_1 = -0.5°, \theta_2 = 5.5°$. By definition, the two targets are resolved in a given run if both $\max_{i=1,2} |\hat{\theta}_i - \theta_i|$ are smaller than $|\theta_2 - \theta_1|/2$, where $\hat{\theta}_i$ denotes the estimation of true direction θ_i. As we can see from Fig. 4 that all methods exhibit a 100% correct target resolution at high SNR region, and the proposed method has higher capability in resolving closely spaced targets compared with the l_1-SVD and RV l_1-SVD algorithms.

Figure 5 shows the bias of the l_1-SVD, RV l_1-SVD algorithms and the proposed method in terms of the angular separation between two targets, where the SNR and the number of snapshots are fixed at 5 dB and 300, respectively. The two uncorrelated targets are considered with DOAs as $\theta_1 = 0°$ and $\theta_2 = 0° + \Delta\theta$, where $\Delta\theta$ varies from 2° to 20°. As indicated in Fig. 5, the bias of the proposed method is lower than the l_1-SVD and RV l_1-SVD algorithms, owing to the virtual aperture being remarkably enlarged. Meanwhile,

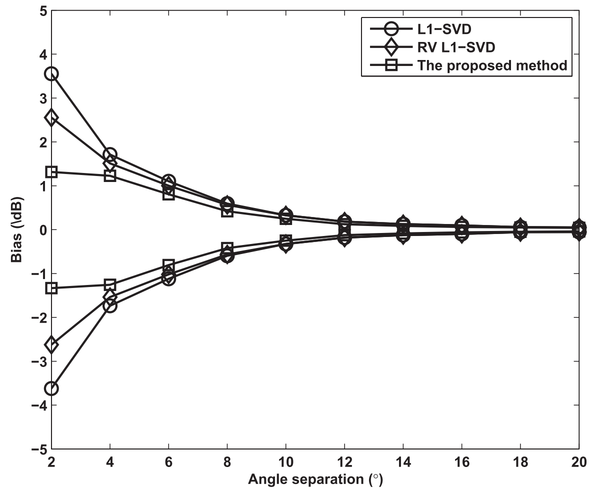

Fig. 5 DOA estimation bias of l_1-SVD, RV l_1-SVD and the proposed method versus angular separation.

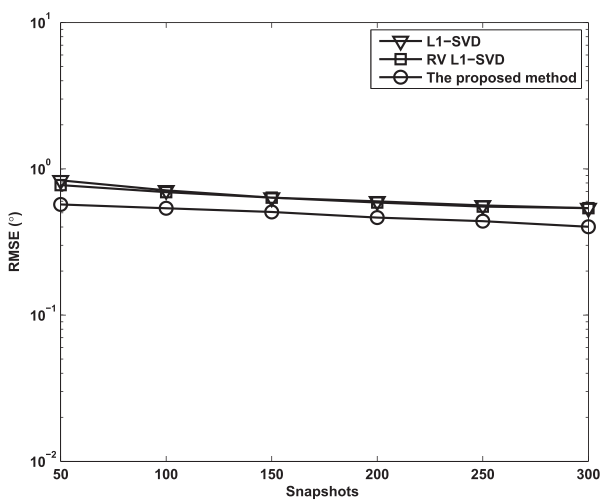

Fig. 6 RMSE versus snapshots for two uncorrelated targets.

all the methods tend to become unbiased when the angular separation is more than 14° apart.

Figure 6 shows the angle estimation performance of the l_1-SVD, RV l_1-SVD algorithms and the proposed method versus snapshots, where two uncorrelated targets are located at $\theta_1 = -0.5°$, and $\theta_1 = 10.5°$ and the SNR is fixed at 0 dB. In Fig. 6, it can be concluded that the proposed method provides better angle estimation performance than the l_1-SVD, RV l_1-SVD algorithms with snapshots increasing.

6. Conclusion

In this paper, we have proposed real-valued reweighted l_1 norm minimization method-based data reconstruction for monostatic multiple-input multiple-output (MIMO) radar. The proposed method only requires a one-dimensional real-valued dictionary for recovering the sparse signal by utilizing the data reconstruction approach and unitary transformation technique. Then a weight matrix is formulated to enhance the solution of l_1 norm penalty. The proposed method provides better angle estimation performance and higher target resolution probability than both of l_1-SVD and RV l_1-SVD algorithms.

References

[1] H. Krim and M. Viberg, “Two decades of array signal processing research: the parametric approach,” IEEE Signal Process. Mag., vol.13, no.4, pp.67–94, 1996.

[2] E. Fishler, A. Haimovich, R. Blum, D. Chizhik, L. Cimini, and R. Valenzuela, “MIMO radar: An idea whose time has come,” Proc. 2004 IEEE Radar Conference (IEEE Cat. No.04CH37509), pp.71–78, 2004.

[3] A. Haimovich, R. Blum, and L. Cimini, “MIMO radar with widely separated antennas,” IEEE Signal Process. Mag., vol.25, no.1, pp.116–129, 2008.

[4] P. Huang, H. Wang, and C. Yang, “Non circular ROOTMUSIC algorithm for monostatic MIMO radar,” IEICE Electron. Express, vol.11, no.13, p.20140318, 2014.

[5] X. Gao, X. Zhang, G. Feng, Z. Wang, and D. Xu, “On the MUSIC-derived Approaches of Angle Estimation for Bistatic MIMO Radar,” Proc. 2009 International Conference on Wireless Networks and Information Systems, pp.343–346, 2009.

[6] D. Xu, X. Zhang, Y. Huang, C. Chen, and J. Li, “Reduced-complexity Capon for direction of arrival estimation in a monostatic multiple-input multiple-output radar,” IET Radar, Sonar & Navigation, vol.6, no.8, pp.796–801, 2012.

[7] C. Duofang, C. Baixiao, and Q. Guodong, “Angle estimation using ESPRIT in MIMO radar,” Electron. Lett., vol.44, no.12, pp.770–771, 2008.

[8] X. Zhang and D. Xu, “Low-complexity ESPRIT-based DOA estimation for colocated MIMO radar using reduced-dimension transformation,” Electron. Lett., vol.47, no.4, pp.283–284, 2011.

[9] W. Wang, X.-P. Wang, Y.-H. Ma, and X. Li, “Conjugate unitary ESPRIT algorithm for bistatic MIMO radar,” IEICE Trans. Electron., vol.E96-C, no.1, pp.124–126, Jan. 2013.

[10] D. Malioutov, M. Cetin, and A.S. Willsky, “A sparse signal reconstruction perspective for source localization with sensor arrays,” IEEE Trans. Signal Process., vol.53, no.8, pp.3010–3022, 2005.

[11] J. Yin and T. Chen, “Direction-of-arrival estimation using a sparse representation of array covariance vectors,” IEEE Trans. Signal Process., vol.59, no.9, pp.4489–4493, 2011.

[12] Z.-Q. He, Z.-P. Shi, and L. Huang, “Covariance sparsity-aware DOA estimation for nonuniform noise,” Digit. Signal. Process., vol.28, pp.75–81, 2014.

[13] Y. Tian, X. Sun, and S. Zhao, “DOA and power estimation using a sparse representation of second-order statistics vector and l_0-norm approximation,” Signal. Process., vol.105, pp.98–108, 2014.

[14] J. Dai, X. Xu, and D. Zhao, “Direction-of-arrival estimation via real-valued sparse representation,” Antennas Wirel. Propag. Lett., vol.12, pp.376–379, 2013.

[15] J. Li, X. Zhang, W. Chen, and T. Hu, “Reduced-dimensional ESPRIT for direction finding in monostatic MIMO radar with double parallel uniform linear arrays,” Wireless. Pers. Commun., vol.77, no.1, pp.1–19, 2014.

[16] W. Wang, X. Wang, H. Song, and Y. Ma, “Conjugate ESPRIT for DOA estimation in monostatic MIMO radar,” Signal. Process., vol.93, no.7, pp.2070–2075, 2013.

[17] X. Wang, W. Wang, J. Liu, X. Li, and J. Wang, “A sparse representation scheme for angle estimation in monostatic MIMO radar,” Signal. Process., vol.104, pp.258–263, 2014.

[18] M. Haardt and J.A. Nossek, “Unitary ESPRIT: How to obtain increased estimation accuracy with a reduced computational burden,” IEEE Trans. Signal Process., vol.43, no.5, pp.1232–1242, 1995.

[19] N. Yilmazer, J. Koh, and T.K. Sarkar, “Utilization of a unitary trans-

form for efficient computation in the matrix pencil method to find the direction of arrival," IEEE Trans. Antennas Propag., vol.54, no.1, pp.175–181, 2006.

[20] J.F. Sturm, "Using SeDuMi 1.02, a Matlab toolbox for optimization over symmetric cones," Optimization Methods and Software, vol.11, no.1-4, pp.625–653, 1999.

[21] M. Grant and S. Boyd, "CVX: MATLAB software for Disciplined Convex Programming, version 1.22," 2012.

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