DOA Estimation by Two-Dimensional Interpolation in the Presence of Mutual Coupling

Qi Liu
Department of Electrical and Computer Engineering
National University of Singapore
Singapore
elelqi@nus.edu.sg

Hui Cao
School of Information Engineering
Wuhan University of Technology
Wuhan, China
iehuicao@whut.edu.cn

Yuntao Wu
School of Computer Science and Engineering
Wuhan Institute of Technology
Wuhan, China
ytwu@sina.com

Changhai Huang
Merchant Marine College
Shanghai Maritime University
Shanghai, China
chhuang@shmtu.edu.cn

***Abstract*—In mobile communication, mutual coupling between array elements will affect the estimation performance of the existing direction of arrival (DOA) estimators. To tackle this problem, an accurate DOA estimation method for coprime array is developed, where the effect of mutual coupling is eliminated by the banded complex symmetric Toeplitz matrix, and further weakened via two-dimensional (2-D) interpolation. The accuracy of the proposed algorithm is verified by the simulation results in comparison of several DOA estimation approaches as well as Cramér-Rao lower bound (CRLB).**

***Index Terms*—Direction of arrival estimation, mutual coupling, array signal processing, coprime array, two-dimensional interpolation**

I. Introduction

Direction of arrival (DOA) estimation has been an important issue in practical mobile communication system. Since all parameters of sources that transmitted by user equipments (UEs) including source types, DOA and power, play a significant role in safeguarding system performance, accurate DOA estimation is essential for the base station to perform downlink precoding or beamforming. Concrete applications can be found in the areas of radar, wireless communication and biomedical imaging [1]–[3]. In the past few decades, numerous DOA estimation algorithms have been proposed for different types of arrays, such as uniform circular array (UCA), uniform rectangular array (URA), non-uniform linear array (NULA), to name just a few. Recently, coprime array has attracted increasing attentions as it can achieve large degrees of freedom (DOF) with large inter-element spacing [4]–[7]. On the basis of coprime array, some DOA estimators are developed in literature. In [8], DECOM is proposed with combined MUSIC for coprime array, which is superior to conventional DOA estimation methods. However, the peak search in [8] is computationally demanding. Hence, to achieve a search-free DOA algorithm, [9] is designed for coprime arrays, where the phase ambiguities are eliminated using a projection-like method.

Nonetheless, in practice, as antenna arrays in cognitive radio systems may be distorted by impairments, e.g., mutual coupling, they will seriously degrade the performance of DOA estimators in users localization [10], [11]. The effect of mutual coupling is first considered in [12]. In the blind calibration, [10] is proposed with the special structure of mutual coupling matrix (MCM) and subspace principle to provide accurate estimation. For eliminating the effect of the unknown mutual coupling, the banded symmetric Toeplitz structure of the MCM in both of the transmit and receive arrays to eliminate the unknown mutual coupling [13], [14]. Similarly, a robust sparse bayesian learning algorithm is proposed in [15], where the covariances of unknown nonuniform noise are updated by using the least squares (LS) strategy.

In this paper, by employing two smaller overlapping subarrays from the signal subspaces, a rotational invariance formulation is constructed to recover the array manifold matrix. Each DOA is estimated by minimizing a nonlinear least squares (NLS) fitting criterion which is solved using Newton's method. As MCM multiplied by the array steering matrix has the same character with the initial steering matrix [10], the ambiguous DOA estimation method is directly applied to estimate the DOAs when mutual coupling is present. In addition, two-dimensional (2-D) interpolation is utilized to further weaken the influence of mutual coupling.

II. Signal Model

In a cognitive radio system, as shown in Fig.1, a secondary network shares the same spectrum with a primary network. Suppose that we have one secondary base station (BS-S), equipped with a ULA of M omnidirectional antenna elements. The secondary network coexists with a primary network

This work was supported in part by the Fundamental Research Funds for Central Universities under Grant WUT: 2018 IVA 097, and in part by the National Natural Science Foundation of China under Project 51909156 and Project 61771353.

Authorized licensed use limited to: National University of Singapore. Downloaded on June 16,2020 at 00:46:09 UTC from IEEE Xplore. Restrictions apply.

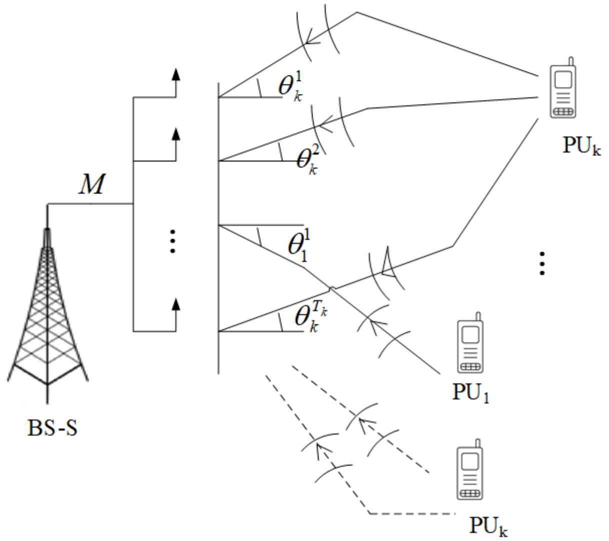

Fig. 1. Generic system model.

consisting of K primary users (PU) with a single receiving antenna. In this work, the BS-S has no information about the channels between the BS-S and the PU. The goal of the cognitive radio system is to provide communications among secondary users, without causing interference to the primary system. Then, the components of the received signal at the BS-S can be modeled as:

$$\mathbf{x}(l) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(l) + \mathbf{n}(l). \tag{1}$$

Assume that there are K far-field sources, then the output of the array is expressed as:

$$\mathbf{x}(l) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_K)]\mathbf{s}(l) + \mathbf{n}(l), \tag{2}$$

where θ_k is the DOA of kth target with respect to the array normal. $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^T$ and $\mathbf{n}(t)$ are source and additive white Gaussian noise (AWGN) vectors, respectively. We consider a coprime array with two uniform linear subarrays [8] under the assumption of two subarrays with inter-element spacing $\frac{M\lambda}{2}$ and $\frac{N\lambda}{2}$, respectively. Therefore, the corresponding steering vector is [16], [17]:

$$\mathbf{a}(\theta_k) = \Big[1, e^{-jM\pi\sin(\theta_k)}, \cdots, e^{-j(N-1)M\sin(\theta_k)}, \\ e^{-jN\pi\sin(\theta_k)}, \cdots, e^{-j(M-1)N\sin(\theta_k)}\Big]. \tag{3}$$

Hence, (2) is rewritten as:

$$\mathbf{x}(l) = [\mathbf{A}_1(\boldsymbol{\theta})\ \mathbf{A}_2(\boldsymbol{\theta})]\mathbf{s}(l) + \mathbf{n}(l), \tag{4}$$

in which $\mathbf{A}_1(\boldsymbol{\theta}) = \big[1, e^{-jM\pi\sin(\theta_k)}, \cdots, e^{-j(N-1)M\sin(\theta_k)}\big]$, and $\mathbf{A}_2(\boldsymbol{\theta}) = \big[e^{-jN\pi\sin(\theta_k)}, \cdots, e^{-j(M-1)N\sin(\theta_k)}\big]$.

III. DOA ESTIMATION

A. Ambiguous DOA Estimation

The covariance matrix of the received signal is computed by

$$\mathbf{R} = \frac{1}{L}\sum_{l=1}^{L}\mathbf{x}(l)\mathbf{x}^T(l), \tag{5}$$

where L is the snapshot number. The eigenvalue decomposition of $\mathbf{R}$ is:

$$\mathbf{R} = \mathbf{U}_s\Lambda_s\mathbf{U}_s^H + \mathbf{U}_n\Lambda_n\mathbf{U}_n^H \tag{6}$$

wherein the K eigenvectors of $\mathbf{R}$ corresponding to the K largest eigenvalues from the signal subspace $\mathbf{U}_s$, whose first N rows and last $M-1$ rows are denoted as $\mathbf{U}_{sN}$ and $\mathbf{U}_{sM}$, respectively. $\mathbf{U}_{sN}$ is corresponding to subarray 1 and $\mathbf{U}_{sM}$ is corresponding to subarray 2. Since $\mathbf{U}_{sN}$ and $\mathbf{A}_1(\boldsymbol{\theta})$ span the same subspace, there exists a nonsingular matrix $\mathbf{T}_1$, such that [18]:

$$\mathbf{A}_1(\boldsymbol{\theta}) = \mathbf{U}_{sN}\mathbf{T}_1. \tag{7}$$

Then $\overline{\mathbf{A}}_{11}(\boldsymbol{\theta}) = \overline{\mathbf{U}}_{sN1}\mathbf{T}_1$, and $\underline{\mathbf{A}}_{11}(\boldsymbol{\theta}) = \underline{\mathbf{U}}_{sN1}\mathbf{T}_1$, where $\overline{\mathbf{A}}_{11}(\boldsymbol{\theta}) = \overline{\mathbf{J}}_1\mathbf{A}_1$ and $\underline{\mathbf{A}}_{11}(\boldsymbol{\theta}) = \underline{\mathbf{J}}_1\mathbf{A}_1$, $\overline{\mathbf{U}}_{sN1}(\boldsymbol{\theta}) = \overline{\mathbf{J}}_1\mathbf{U}_{sN}$ and $\underline{\mathbf{U}}_{sN1}(\boldsymbol{\theta}) = \underline{\mathbf{J}}_1\mathbf{U}_{sN}$, with $\overline{\mathbf{J}}_1 = [\mathbf{I}_{N-1}\ \mathbf{0}_{(N-1)\times 1}]$, and $\underline{\mathbf{J}}_1 = [\mathbf{0}_{(N-1)\times 1}\ \mathbf{I}_{N-1}]$. As the shift invariance in ESPRIT [18] can be described as:

$$\underline{\mathbf{A}}_{11}(\boldsymbol{\theta}) = \overline{\mathbf{A}}_{11}(\boldsymbol{\theta})\Phi_1, \tag{8}$$

where the Vandermonde structure matrix $\Phi = \mathrm{diag}(e^{-jM\pi\sin(\theta_1)}, \cdots, e^{-jM\pi\sin(\theta_K)})$ with $\mathrm{diag}(\cdot)$ representing a diagonal matrix, the rotational invariance equation (RIE) is:

$$\underline{\mathbf{U}}_{sN1} = \overline{\mathbf{U}}_{sN1}\Psi_1. \tag{9}$$

with $\Psi_1 = \mathbf{T}_1\Phi_1\mathbf{T}_1^{-1}$. Based on [18], $\mathbf{A}_1(\boldsymbol{\theta})$ in (7) is identifiable. Assume that there exists an estimate of $\mathbf{T}_1$, denoted as $\hat{\mathbf{T}}_1$, such that $\mathbf{A}_1(\boldsymbol{\theta})$ can be recovered by multiplying the signal subspace with $\mathbf{T}_1$. Then the estimate of $\mathbf{A}_1(\boldsymbol{\theta})$, denoted as $\hat{\mathbf{A}}_1(\boldsymbol{\theta})$, is:

$$\hat{\mathbf{A}}_1(\boldsymbol{\theta}) = \hat{\mathbf{U}}_{sN1}\hat{\mathbf{T}}_1. \tag{10}$$

Note that in the presence of noise, $\hat{\mathbf{A}}_1(\boldsymbol{\theta})$ in (10) is not exactly equal to $\mathbf{A}_1(\boldsymbol{\theta})$. That is, $\hat{\mathbf{A}}_1(\boldsymbol{\theta}) = \mathbf{A}_1(\boldsymbol{\theta})\Gamma\Sigma + \Delta\mathbf{A}$ with a permutation matrix Γ, a diagonal scaling matrix $\Sigma = \mathrm{diag}(\gamma_1, \cdots, \gamma_K)$ and an error term $\Delta\mathbf{A}$. Let the element of $\hat{\mathbf{A}}_1(\boldsymbol{\theta})$ be $\hat{\mathbf{a}}_k$. Then,

$$\min_{\theta_k,\gamma_k} ||\hat{\mathbf{a}}_k - \gamma_k\mathbf{a}(\theta_k)||_2^2,\ \forall k = 1, \cdots, K. \tag{11}$$

This is a single-tone parameter estimation problem which can be solved using the given measurement $\hat{\mathbf{a}}_k$ to find the angle θ_k and amplitude γ_k. To solve this problem, the Newton's method is utilized. As γ_k is deterministic, by substituting $\hat{\gamma}_k = (\mathbf{a}(\theta_k))^\dagger\hat{\mathbf{a}}_k$ into (11), we obtain the ML formulation with respect to θ_k [19]:

$$\hat{\theta}_k = \arg\max_{\theta_k} |\mathbf{a}^H(\theta_k)\hat{\mathbf{a}}_k|^2,\ \forall k = 1, \cdots, K. \tag{12}$$

As $\mathbf{a}^H(\theta_k)\mathbf{a}(\theta_k) = N$ is constant, it is neglected. Hence, θ_k is updated by

$$\theta_k^{(l+1)} = \theta_k^{(l)} - \mu^{(l)}h^{-1}(\theta_k^{(l)})g(\theta_k^{(l)}), \tag{13}$$

where $\mu^{(l)}$ is the step-size and

$$g(\theta_k) = 2\Re(\dot{\mathbf{a}}_k^H\hat{\mathbf{a}}_k\mathbf{a}_k^H\hat{\mathbf{a}}_k) \tag{14}$$

Authorized licensed use limited to: National University of Singapore. Downloaded on June 16,2020 at 00:46:09 UTC from IEEE Xplore. Restrictions apply.

$$h(\theta_k) = 2\Re(\ddot{\mathbf{a}}_k^H \hat{\mathbf{a}}_k \hat{\mathbf{a}}_k^H \mathbf{a}_k + \dot{\mathbf{a}}_k^H \hat{\mathbf{a}}_k \hat{\mathbf{a}}_k^H \dot{\mathbf{a}}_k) \quad (15)$$

are gradient and Hessian, respectively. $\Re$ denotes the real part, $\dot{\mathbf{a}}_k = \frac{\partial \mathbf{a}(\theta_k)}{\partial \theta_k}$, and $\ddot{\mathbf{a}}_k = \frac{\partial^2 \mathbf{a}(\theta_k)}{\partial \theta_k^2}$.

B. Unique DOA Determination

Recall that the estimate of DOA in III-A is $\hat{\nu}_k = M\pi \sin(\hat{\theta}_k)$. On the basis of coprime array, DOA is actually estimated from the first N rows of $\mathbf{U}_{sM}$, termed as $\hat{\theta}_{k,M}$. Then, we have:

$$\sin \hat{\theta}_{k,M} = \hat{\nu}_k/(M\pi), k = 1, \cdots, K. \quad (16)$$

We need to recover all estimation results based on the ambiguous estimations. According to the idea in [8], all the M estimations are obtained, but the correct estimation should be obtained by finding the coincide one or the average of two nearest ones from the M estimations. Following the same procedure from (7) to (16), we utilize the subspace $\mathbf{U}_{sM}$ of the second subarray to obtain the estimated DOA from the last $M-1$ rows, denoted as $\hat{\theta}_{k,N}$. Then,

$$\sin \hat{\theta}_{k,N} = \hat{\eta}_k/(N\pi), k = 1, \cdots, K. \quad (17)$$

We conclude that the DOA is finally estimated by

$$\hat{\theta}_k = \frac{\hat{\theta}_{k,N} + \hat{\theta}_{k,M}}{2}. \quad (18)$$

C. DOA Estimation with Mutual Coupling

In practice, interactions between the elements of the array will result in mutual coupling, as shown in Figs. 2 and 3, which distorts the ideal steering vector significantly. In this case, the true steering vector is modified as:

$$\mathbf{a}_m(\theta_k) = \mathbf{C}\mathbf{a}(\theta_k), \quad (19)$$

where $\mathbf{C}$ is the MCM with symmetric Toeplitz form:

$$\mathbf{C} = \begin{bmatrix} 1 & c_1 & \cdots & c_{p-1} & \cdots & c_{P-1} \\ c_1 & 1 & c_1 & \cdots & \ddots & \vdots \\ \vdots & c_1 & 1 & c_1 & \cdots & c_{p-1} \\ c_{p-1} & \cdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \cdots & c_1 & 1 & c_1 \\ c_{P-1} & \cdots & c_{p-1} & \cdots & c_1 & 1 \end{bmatrix}_{P \times P}, \quad (20)$$

and $P = M + N - 1$, $c_i = \rho_i e^{j\phi_i} (i = 1, 2, \cdots, M + N - 2)$ is the mutual coupling coefficient with the amplitude ρ_i and the phase ϕ_i.

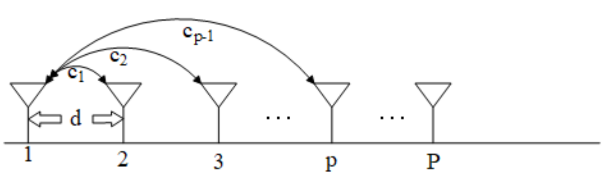

Fig. 2. The effect of mutual coupling.

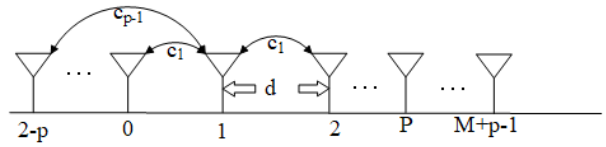

Fig. 3. The effect while adding auxiliary arrays.

When two sensors are far from each other, the mutual coupling coefficients between them can be approximated as zero. Hence, the ULA coupling model is often considered to have just finite nonzero coefficients. In the existing methods [13], [14], a banded symmetric Toeplitz matrix is used to eliminate the effect of the mutual coupling. Based on this model, we assume that there are p nonzero mutual coupling coefficients. That is to say, for the ith sensor, coupling comes from the $(i-p+1)$th, ..., $(i-1)$th, $(i+1)$th, ..., $(i+p-1)$th sensors. Then the output of the $M+N-1$-element array is written as:

$$\mathbf{y}(l) = \mathbf{C}\mathbf{x}(l) = \mathbf{C}\mathbf{A}(\boldsymbol{\theta})\mathbf{s}(l) + \mathbf{n}(l), \quad (21)$$

where $\mathbf{C}$ is the MCM with updated form as:

$$\mathbf{C} = \begin{bmatrix} 1 & c_1 & \cdots & c_{p-1} & 0 & 0 \\ c_1 & 1 & c_1 & \cdots & \ddots & 0 \\ \vdots & c_1 & 1 & c_1 & \cdots & c_{p-1} \\ c_{p-1} & \cdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \cdots & c_1 & 1 & c_1 \\ 0 & 0 & c_{p-1} & \cdots & c_1 & 1 \end{bmatrix}_{P \times P}. \quad (22)$$

The spatial covariance matrix of $\mathbf{y}(l)$ is presented as:

$$\mathbf{R}_\mathbf{y} = \frac{1}{L}\sum_{l=1}^{L} \mathbf{y}(l)\mathbf{y}^T(l), \quad (23)$$

Performing the eigenvalue decomposition on $\mathbf{R}_\mathbf{y}$:

$$\mathbf{R}_\mathbf{y} = \mathbf{U}_{\mathbf{y}s}\Lambda_{\mathbf{y}s}\mathbf{U}_{\mathbf{y}s}^H + \mathbf{U}_{\mathbf{y}n}\Lambda_{\mathbf{y}n}\mathbf{U}_{\mathbf{y}n}^H, \quad (24)$$

then we have [10]:

$$\mathrm{span}\{\mathbf{C}\mathbf{A}(\theta)\} = \mathrm{span}\{\mathbf{U}_{\mathbf{y}s}\}. \quad (25)$$

As the MCM multiplied the array steering matrix has the same character with the initial steering matrix, the DOA estimation algorithm in Section-III can be applied to estimate the DOA in mutual coupling signals with the use of subspace $\mathbf{U}_{\mathbf{y}s}$. To further eliminate the mutual coupling effect on DOA estimator and simplify the computational complexity of the covariance matrix, four sub-covariance matrices on $\mathbf{R}_\mathbf{y}$ are constructed as:

$$\begin{aligned} \mathbf{R}_1 &= \mathbf{R}_\mathbf{y}(1 : M+N-2, 1 : M+N-2), \quad (26) \\ \mathbf{R}_2 &= \mathbf{R}_\mathbf{y}(1 : M+N-2, 2 : M+N-1), \\ \mathbf{R}_3 &= \mathbf{R}_\mathbf{y}(2 : M+N-1, 1 : M+N-2), \\ \mathbf{R}_4 &= \mathbf{R}_\mathbf{y}(2 : M+N-1, 2 : M+N-1). \end{aligned}$$

Authorized licensed use limited to: National University of Singapore. Downloaded on June 16,2020 at 00:46:09 UTC from IEEE Xplore. Restrictions apply.

Then we perform the eigenvalue decomposition on the four constructed matrices to obtain their subspaces for DOA estimation using the procedure from (6) to (18). To investigate the performance, we compare the mean square error (MSE) of the proposed algorithm with the use of $\mathbf{R_y}$, $\mathbf{R}_1$, $\mathbf{R}_2$, $\mathbf{R}_3$, and $\mathbf{R}_4$, respectively. The MSE is defined as:

$$\mathrm{MSE} = \frac{1}{KQ}\sum_{k=1}^{K}\sum_{q=1}^{Q}(\hat{\theta}_{k,q} - \theta_k)^2, \tag{27}$$

where $\hat{\theta}_{k,q}$ denotes the estimation of DOA of kth signal in the qth Monte-Carlo round. Q is the number of Monte-Carlo runs. We use $Q = 200$ in the simulation. As shown in Fig. 4, it is observed that the MSE of using $\mathbf{R}_3$ or $\mathbf{R}_4$ outperforms that with $\mathbf{R}_1$ or $\mathbf{R}_2$, respectively. Moreover, the MSEs of $\mathbf{R}_3$ and $\mathbf{R}_4$ enjoy comparable performance with that by $\mathbf{R_y}$. Therefore, it is intuitive to average DOA only from the estimates of $\mathbf{R}_3$ and $\mathbf{R}_4$, namely,

$$\hat{\theta}_k = \frac{\hat{\theta}_{k,\mathbf{R}_3} + \hat{\theta}_{k,\mathbf{R}_4}}{2}. \tag{28}$$

where $\hat{\theta}_{k,\mathbf{R}_3}$ and $\hat{\theta}_{k,\mathbf{R}_4}$ denote the DOA estimates based on $\mathbf{R}_3$ and $\mathbf{R}_4$, respectively.

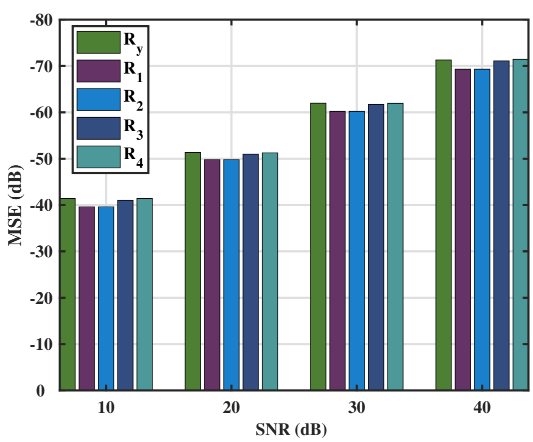

Fig. 4. MSE versus SNR.

IV. Numerical results

In this section, computer simulations are conducted to evaluate the performance of the proposed algorithm, where the coprime array sizes are $M = 7$, $N = 5$, and the number of snapshots is $L = 10$. All results comes from the average of 200 Monte-Carlo runs, and are performed on the MATLAB R2017b of 64-bit Windows 10 operating system with 1.70 GHz Intel Xeon CPU E5-2609 and 32 GB RAM.

In the first test, the DOA is assigned at $8°$. The number of nonzero mutual coupling coefficient is $P = 1$ and $c = e^{-j\pi\frac{35}{180}}$. As shown in Fig. 5, all methods can approach the Cramér-Rao lower bound (CRLB) when the signal-to-noise ratio (SNR) is larger than 10 dB, except the method in [15] with the SNR threshold larger than 15 dB. The proposed method performs the best performance among all methods in comparison.

To further investigate the performance of the proposed algorithm, the case of multiple sources is considered with DOA = $[3° \; 8°]$. The number of nonzero mutual coupling coefficient is $P = 2$ and $c = [e^{-j\pi\frac{10}{180}} \; e^{-j\pi\frac{45}{180}}]$. The corresponding result is plotted in Fig. 6. We can see that the performance of the proposed method and [14] are comparable, and both of them are superior than [15].

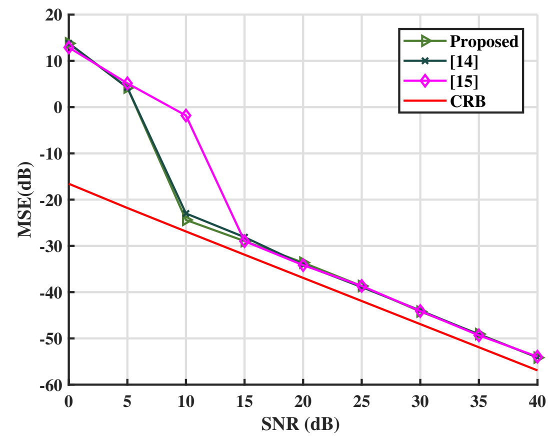

Fig. 5. MSE versus SNR for DOA=8°.

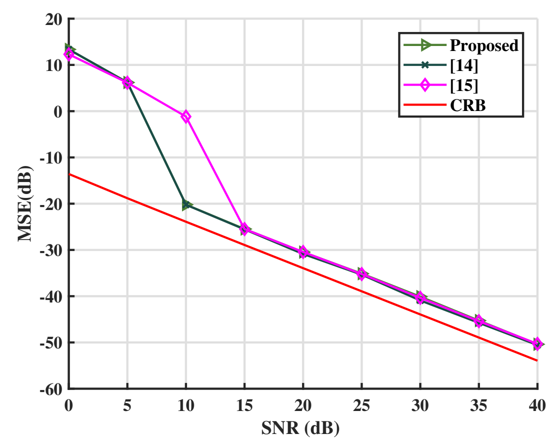

Fig. 6. MSE versus SNR for DOA=[3° 8°].

V. Conclusion

In this paper, the problem of DOA estimation on the coprime array is addressed, where the ambiguous DOA estimator is developed at first without mutual coupling. Then, with the help of the banded complex symmetric Toeplitz structure to eliminate the effect of mutual coupling, the ambiguous DOA estimator is applied as the solver. Therein 2-D interpolation based on covariance matrix is utilized to further weaken the influence of mutual coupling. Numerical results have been conducted to verify the effectiveness of the proposed algorithm.

Authorized licensed use limited to: National University of Singapore. Downloaded on June 16,2020 at 00:46:09 UTC from IEEE Xplore. Restrictions apply.

REFERENCES

[1] H. Li, Y. Li, and Z. He, "MIMO radar clutter mitigation based on joint beam forming and joint domain localized processing," *EURASIP J. Wireless Commun. Networking*, vol. 2013, no. 1, pp. 1–10, 2013.

[2] Q. Liu, H. So, and Y. Gu, "Off-grid DOA estimation with nonconvex regularization via joint sparse representation," *Signal Process.*, vol. 140, pp. 171–176, Nov. 2017.

[3] Q. Liu, Y. Gu, and H. So., "DOA estimation in impulsive noise via low-rank matrix approximation and weakly convex optimization," *IEEE Trans. Aerospace and Electronic Syst.*, vol. 55, no. 6, pp. 3603–3616, Dec. 2019.

[4] P. Vaidyanathan and P. Pal, "Sparse sensing with co-prime samplers and arrays," *IEEE Trans. Signal Process.*, vol. 59, no. 2, pp. 573–586, Feb. 2011.

[5] ——, "Theory of sparse coprime sensing in multiple dimensions," *IEEE Trans. Signal Process.*, vol. 59, no. 8, pp. 3592–3608, Aug. 2011.

[6] C. Zhou, Y. Gu, Z. Shi, and Y. D. Zhang, "Off-grid direction-of-arrival estimation using coprime array interpolation," *IEEE Signal Process. Lett.*, vol. 25, no. 11, pp. 1710–1714, Nov. 2018.

[7] C. Zhou, Y. Gu, X. Fan, Z. Shi, G. Mao, and Y. D. Zhang, "Direction-of-arrival estimation for coprime array via virtual array interpolation," *IEEE Trans. Signal Process.*, vol. 66, no. 22, pp. 5956–5971, Nov. 2018.

[8] C. Zhou, Z. Shi, Y. Gu, and X. Shen, "DECOM: DOA estimation with combined MUSIC for coprime array," in *2013 International Conference on Wireless Communications and Signal Processing*, Oct. 2013, pp. 1–5.

[9] Z. Weng and P. M. Djurić, "A search-free DOA estimation algorithm for coprime arrays," *Digital Signal Process.*, vol. 24, pp. 27–33, Oct. 2014.

[10] M. Lin and L. Yang, "Blind calibration and DOA estimation with uniform circular arrays in the presence of mutual coupling," *IEEE Antennas Wireless Propag. Lett.*, vol. 5, pp. 315–318, 2006.

[11] Li Hao and Wei Ping, "DOA estimation in an antenna array with mutual coupling based on ESPRIT," in *2013 International Workshop on Microwave and Millimeter Wave Circuits and System Technology*, Oct. 2013, pp. 86–89.

[12] I. J. Gupta and A. K. Ksienski, "Effect of mutual coupling on the performance of adaptive arrays," *IEEE Trans. Antennas Propag.*, vol. 31, no. 5, pp. 785–791, Sep. 1983.

[13] X. Wang, W. Wang, J. Liu, Q. Liu, and B. Wang, "Tensor-based real-valued subspace approach for angle estimation in bistatic MIMO radar with unknown mutual coupling," *Signal Process.*, vol. 116, pp. 152 – 158, 2015.

[14] J. Liu, X. Wang, and W. Zhou, "Covariance vector sparsity-aware DOA estimation for monostatic MIMO radar with unknown mutual coupling," *Signal Process.*, vol. 119, pp. 21 – 27, 2016.

[15] X. Wang, M. Huang, C. Shen, C. Cao, K. Zhang, and G. Bi, "Robust DOA estimation for MIMO radar in unknown nonuniform noise and mutual coupling," in *2018 IEEE 10th Sensor Array and Multichannel Signal Processing Workshop (SAM)*, July 2018, pp. 194–197.

[16] Z. Weng and P. M.Djurić, "A search-free DOA estimation algorithm for coprime arrays," *Digital Signal Process.*, vol. 10, pp. 142–149, 2008.

[17] J. Li, M. Shen, and D. Jiang, "DOA estimation based on combined ESPRIT for co-prime array," in *2016 IEEE 5th Asia-Pacific Conference on Antennas and Propagation (APCAP)*, Jul. 2016, pp. 117–118.

[18] C. Qian, "A simple modification of ESPRIT," *IEEE Signal Process. Lett.*, vol. 25, no. 8, pp. 1256–1260, Aug. 2018.

[19] D. C. Rife and R. R. Boorstyn, "Single tone parameter estimation from discrete-time observations," *IEEE Trans. Inf. Theory*, vol. 20, no. 5, pp. 591–598, Sep. 1974.

Authorized licensed use limited to: National University of Singapore. Downloaded on June 16,2020 at 00:46:09 UTC from IEEE Xplore. Restrictions apply.