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Direction of arrival estimation via reweighted l_1 norm penalty algorithm for monostatic MIMO radar

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Abstract In this paper, a reweighted l_1 norm penalty algorithm for direction of arrival (DOA) estimation in monostatic multiple-input multiple-output radar is proposed. In the proposed method, exploiting the inherent multidimensional structure of received data after matched filtering, the singular value decomposition (SVD) technique of the data matrix is employed to reduce the dimension of the received signal. Then a novel weight matrix is designed for reweighting the l_1 norm minimization by exploiting the coefficients of the reduced-dimensional Capon (RD-Capon) spatial spectrum. The proposed algorithm enhances the sparsity of the solution by the reweighted l_1 norm constraint minimization, and the DOAs can be estimated by finding the non-zero rows of the recovered matrix. Owing to utilizing the SVD technique and the novel weight matrix, the proposed algorithm can provide better angle estimation performance than RD-Capon and l_1-SRACV algorithms. Furthermore, it is suitable for coherent sources and has a low sensitivity to the incorrect determination of the source numbers. The effectiveness and superior performance of the proposed algorithm are demonstrated by numerical simulations.

Keywords MIMO radar · DOA estimation · Sparse representation · High-resolution · Reweighted l_1 norm

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1 Introduction

Direction of arrival (DOA) estimation of far-field narrowband signal has been drawn considerable attention in the past few decades (Krim and Viberg 1996). Multiple-input multiple-output (MIMO) radar uses multiple antennas to simultaneously transmit diverse waveforms instead of coherent waveforms and utilizes multiple antennas to receive the reflected signals. It has been verified that MIMO radar has a lot of potential advantages over the conventional phased-array radar (Li and Stoica 2007, 2008; Liao and Chan 2015; Xu et al. 2008; Huang et al. 2013; Zheng and Chen 2015). Many high-resolution algorithms have been developed for MIMO radar such as reduced-dimensional Capon (RD-Capon) (Zhang et al. 2012), conjugate-unitary ESPRIT (CU-ESPRIT) (Wang et al. 2013) and reweighted subspace fitting algorithms (Hu et al. 2012). However, most of methods mentioned above are based on the subspace technique, which will bring about rank loss in the covariance matrix of received data when the sources are coherent, and lead to parameter estimation performance degradation.

Recently, sparse signal representation (SSR) theory has attracted a significant interest in DOA estimation and a lot of algorithms have been proposed in this issue (Donoho et al. 2006; Malioutov et al. 2005; Yin and Chen 2011; He et al. 2014; Candes et al. 2008). The sparse signal representation process can be regarded as a l_0 norm approximation problem. However, it has been pointed in Donoho et al. (2006) that l_0 norm approximation problem is NP-hard, and sensitive to noise. In Malioutov et al. (2005) cast the DOA estimation problem into a sparse signal recovery problem and the l_1-SVD approach is proposed to enforce the sparsity of the solution, in which the SVD technique is used to simplify calculation. Additionally, Yin and Chen use the sparse representation of the array covariance vectors (SRACV) for DOA estimation in Yin and Chen (2011). Although the l_1 norm minimization is a convex problem, an important drawback of the l_1 norm is the undemocratic penalty for large coefficients. To deal with the issue, a lot of iterative reweighted l_1 minimization algorithms (He et al. 2014; Candes et al. 2008) were designed to discourage non-zero entries in the recovered signals. However, the DOA estimation problem in monostatic MIMO radar is usually encountered with MMV problem, and the iterative algorithms are not suitable for MMV problem any more. On the other hand, the methods mentioned above are based on l_1 norm minimization, the sparsest solution of l_1-norm penalty does not approximate to the l_0 norm penalty effectively.

In this paper, in order to better enhance the sparsity of the solution, we present an improved reweighted l_1 norm penalty algorithm for direction of arrival (DOA) estimation in monostatic MIMO radar. The proposed algorithm includes three steps: (i) utilize the SVD technique to reduce the computational complexity of the sparse reconstruction and the sensitivity to noise. (ii) use Lagrange multipliers to solve a constrained estimation problem, then the coefficients of the RD-Capon spatial spectrum are exploited to design a weight matrix for l_1 norm minimization. (iii) formulate a reweighted l_1 norm constraint minimization to enforce the sparsity of the solution for MMV problem, then the DOA estimation is obtained by finding the non-zero elements of the recovered matrix. Due to exploit the SVD technique and the weight matrix, the angle estimation performance of the proposed method is better than RD-Capon and l_1-SRACV algorithms especially in low SNR region. Additionally, the proposed algorithm is capable of handing with coherent sources without requiring any decorrelation operation, and performs well without knowing a priori knowledge of the number of sources.

The reminder of this paper is organized as follows. In Sect. 2, we briefly depict data model. In Sect. 3, the reweighted l_1 norm penalty algorithm for DOA estimation is proposed. The performance analysis of the proposed method and the Cramer–Rao bound are given in Sect. 4. Several simulation results verify the performance of the proposed algorithm in Sect. 5. Conclusions are given in Sect. 6.

Notation: $(\cdot)^H$, $(\cdot)^T$, $(\cdot)^{-1}$, $(\cdot)^*$, $(\cdot)^+$ represent the Hermitian transpose, transpose, inverse, complex conjugate, pseudo inverse, respectively. $\otimes$ denotes the Kronecker operator, and $\odot$ denotes the Khatri–Rao matrix product. $\text{diag}(\cdot)$ and $\text{blkdiag}(\cdot)$ denotes the diagonalization operation and block diagonalization operation, respectively. $\text{Re}(\cdot)$ denotes the real-part operation, and $\text{vec}(\cdot)$ denotes a matrix operation that stacks the columns of a matrix under each other to form a new vector. $\mathbf{I}_M$ and $\mathbf{0}_M$ are a $M \times M$ dimensional unit matrix and a $M \times M$ diagonal matrix with all elements equal 0, respectively. Furthermore, $\|\cdot\|_1$, $\|\cdot\|_F$ and $(\cdot)^{l_2}$ represent the l_1 norm, Frobenius norm and l_2 norm of each row of the matrix, respectively.

2 Data model

Consider a narrow-band monostatic MIMO radar system with M transmit sensors and N receive sensors, and both of transmit and receive array are uniform linear arrays (ULAs), shown in Fig. 1. It is assumed that there are K far-field signals with directions $\theta = [\theta_1, \theta_2, \ldots, \theta_K]$. The inter-element spaces of the transmit and receive arrays are half-wavelength, M transmit antennas emit M different orthogonal narrowband waveforms, which have identical bandwidth and centre frequency. According to Chan et al. (2014), after matched filtering, the output at the receive array can be expressed as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{w}(t) \quad (1)$$

where $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T$ is the $K \times 1$ zero-mean signal vector, $\mathbf{w}(t)$ is the $MN \times 1$ Gaussian white noise vector with zero mean and covariance matrix $\sigma^2\mathbf{I}_{MN} \cdot \mathbf{A}$ is

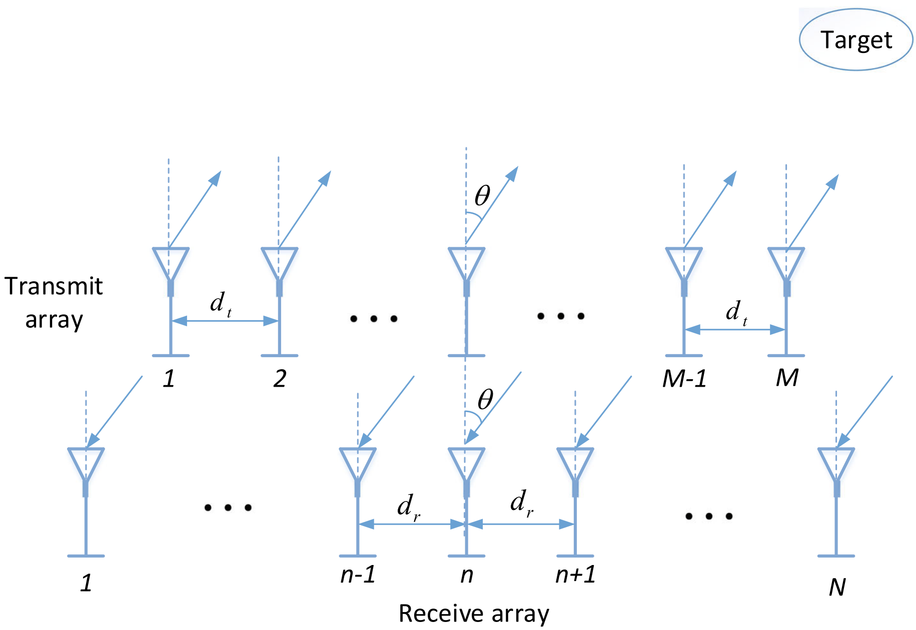

Fig. 1 The configuration of monostatic multiple-input multiple-output (MIMO) radar

an $MN \times K$ array manifold matrix, depending on the unknown DOAs, which can be written as

$$\mathbf{A} = \mathbf{A_1} \odot \mathbf{A_2} \in \mathbb{C}^{MN \times K} \tag{2}$$

$$\mathbf{A_1} = [\mathbf{a}_t(\theta_1), \mathbf{a}_t(\theta_2), \ldots, \mathbf{a}_t(\theta_K)] \in \mathbb{C}^{M \times K} \tag{3}$$

$$\mathbf{A_2} = [\mathbf{a}_r(\theta_1), \mathbf{a}_r(\theta_2), \ldots, \mathbf{a}_r(\theta_K)] \in \mathbb{C}^{N \times K} \tag{4}$$

$$\mathbf{a}_t(\theta_k) = [1, \exp(j\pi \sin\theta_k), \ldots, \exp(j\pi(M-1)\sin\theta_k)]^T \tag{5}$$

$$\mathbf{a}_r(\theta_k) = [1, \exp(j\pi \sin\theta_k), \ldots, \exp(j\pi(N-1)\sin\theta_k)]^T \tag{6}$$

By collecting L snapshots, the received data can be rewritten as

$$\mathbf{X} = \mathbf{AS} + \mathbf{W} \tag{7}$$

where $\mathbf{X} = [\mathbf{x}(t_1), \mathbf{x}(t_2), \ldots, \mathbf{x}(t_L)]$ is a data matrix, $\mathbf{S} = [\mathbf{s}(t_1), \mathbf{s}(t_2), \ldots, \mathbf{s}(t_L)]$ is source waveform matrix and $\mathbf{W} = [\mathbf{w}(t_1), \mathbf{w}(t_2), \ldots, \mathbf{w}(t_L)]$ is sensor noise matrix.

3 Reweighted l_1 norm penalty for DOA estimation

In this section, we firstly formulate the DOA estimation into a sparse signal representation problem in monostatic MIMO radar. In order to reduce both the computational complexity and the sensitivity to noise, the singular value decomposition (SVD) can be used for the received data $\mathbf{X}$, which is shown as

$$\mathbf{X} = \boldsymbol{U}\boldsymbol{\Lambda}\mathbf{V}^H \tag{8}$$

where $\mathbf{U} \in \mathbb{C}^{MN \times MN}, \mathbf{V} \in \mathbb{C}^{L \times L}$ are the left and right singular matrices, respectively, $\boldsymbol{\Lambda} \in \mathbb{C}^{MN \times L}$ is the diagonal singular value matrix with the singular values arranged in descending order.

Defining a $MN \times K$ reduced-dimensional matrix $\mathbf{X_{SV}}$, which multiplies the received signal $\mathbf{X}$ by the first K columns of $\mathbf{U}$, we obtain

$$\mathbf{X_{SV}} = \mathbf{AS_{SV}} + \mathbf{W_{SV}} \tag{9}$$

where $\mathbf{X_{SV}} = \boldsymbol{U}\boldsymbol{\Lambda}\mathbf{D_L} = \mathbf{XVD_L}, \mathbf{D_L} = [\,\mathbf{I_L}\ \mathbf{0}\,]^T$, and $\mathbf{0}$ is a $K \times (T-K)$ zero matrix, $\mathbf{S_{SV}} = \mathbf{SVD_L} \in \mathbb{C}^{K \times K}, \mathbf{W_{SV}} = \mathbf{WVD_L} \in \mathbb{C}^{MN \times K}$.

Let $\{\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_J\}$ be the discrete sampling grid of all potential DOAs, from $-90°$ to $90°$ with $0.01°$ intervals. The number of sampling angles will typically be much greater than the number of sources and the array sensors. i.e. $J \gg K, M$. In the sparse signal recovery framework, the DOA estimation is confined to the grid. If $\{\hat{\theta}_i\}_{i=1}^J$ are dense enough, steering vectors $\{\mathbf{a}_t(\hat{\theta}_k)\}_{k=1}^K$ (or $\{\mathbf{a}_r(\hat{\theta}_k)\}_{k=1}^K$) can be expected to be very close to $\{\mathbf{a}_t(\hat{\theta}_k)\}_{k=1}^K$ (or $\{\mathbf{a}_r(\hat{\theta}_k)\}_{k=1}^K$). Then constructing a two-dimensional transmit–receive dictionary composed of steering vectors corresponding to the potential DOAs.

$$\mathbf{A}_{\hat{\theta}} = \hat{\mathbf{A}}_\mathbf{1} \odot \hat{\mathbf{A}}_\mathbf{2} \in \mathbb{C}^{MN \times J} \tag{10}$$

$$\hat{\mathbf{A}}_\mathbf{1} = [\mathbf{a}_t(\hat{\theta}_1), \mathbf{a}_t(\hat{\theta}_2), \ldots, \mathbf{a}_t(\hat{\theta}_J)] \in \mathbb{C}^{M \times J} \tag{11}$$

$$\hat{\mathbf{A}}_\mathbf{2} = [\mathbf{a}_r(\hat{\theta}_1), \mathbf{a}_r(\hat{\theta}_2), \ldots, \mathbf{a}_r(\hat{\theta}_J)] \in \mathbb{C}^{N \times J} \tag{12}$$

In the SSR framework, $\mathbf{A}_{\hat{\theta}}$ is known and contains all the information of $\mathbf{A}$. If the DOAs are on or close to the discrete sampling grid, under the sparse signal representation framework, the model in Eq. (9) can be rewritten as

$$\mathbf{X}_{\mathbf{SV}} = \mathbf{A}_{\hat{\theta}} \mathbf{S}_{\hat{\theta}} + \mathbf{W}_{\mathbf{SV}} \tag{13}$$

where $\mathbf{S}_{\hat{\theta}}$ is a $J \times K$ sparse matrix whose jth row corresponding to a potential DOA $\hat{\theta}_j$. Since $\mathbf{S}_{\hat{\theta}}$ and $\mathbf{S}_{\mathbf{SV}}$ have the same row support, DOA estimation based on Eq. (13) is equivalent to find a sufficiently sparse of $\mathbf{S}_{\hat{\theta}}$. The sparse recovery process can be regarded as the l_0 norm constrain minimization problem. However, the l_0 norm minimization problem is nonconvex, NP-hard and thereby cannot be solved. The l_1 norm penalty instead of l_0 norm penalty is used to solve the issue (Malioutov et al. 2005). In order to obtain $\mathbf{S}_{\hat{\theta}}$, the constraint minimization problem is considered as following

$$\min_{\mathbf{S}_{\mathbf{SV}}} \left\| \mathbf{S}_{\hat{\theta}}^{(l_2)} \right\|_1 \quad s.t. \quad \left\| \mathbf{X}_{\mathbf{SV}} - \mathbf{A}_{\hat{\theta}} \mathbf{S}_{\hat{\theta}} \right\|_F \leq \tilde{\beta} \tag{14}$$

where $\tilde{\beta}$ is the regularization parameter. After obtaining the recovery matrix, DOA estimation is converted into finding the position of the nonzero entries in $\mathbf{S}_{\hat{\theta}}^{(l_2)}$. In Eq. (14), although the l_1-SVD algorithm can guarantee the convergence to global minima easily, the important drawbacks of the l_1 norm are the undemocratic penalty for large coefficients, and the sparsest solution of l_1-norm penalty does not approximate to the l_0 norm penalty effectively, which results in the degradation of sparse signal reconstruction performance.

Now, in order to deal with these above problems, a reweighted l_1 norm penalty algorithm for the MMV problem is proposed by utilizing coefficients of the RD-Capon spatial spectrum. According to the Zhang and Xu (2010), the RD-Capon function can be obtained by solving the following constrained minimization problem (Zhang and Xu 2010)

$$f(\hat{\theta}) = \arg \max_{\mathbf{e}^T \mathbf{a}_t(\hat{\theta})=1} \mathbf{a}_t(\hat{\theta})^H \mathbf{Q}(\hat{\theta}) \mathbf{a}_t(\hat{\theta}) \tag{15}$$

where $\mathbf{Q}(\hat{\theta}) = [\mathbf{a}_r(\hat{\theta}) \otimes \mathbf{I}_M]^H \mathbf{E}_s \mathbf{E}_s^H [\mathbf{a}_r(\hat{\theta}) \otimes \mathbf{I}_M]$, and $\mathbf{e} = [1, 0, \ldots, 0]^T$. Utilizing Lagrange multipliers to solve the constrained minimization problem, we can obtain the RD-capon spectrum function as

$$f(\hat{\theta}) = \arg \max_{\hat{\theta}} \mathbf{e}^T \mathbf{Q}(\hat{\theta})^{-1} \mathbf{e} \tag{16}$$

Computing the solution of $\mathbf{e}^T \mathbf{Q}(\hat{\theta})^{-1} \mathbf{e}$ over $\theta \in (-90°, 90°)$, the K largest peaks of the conventional RD-Capon for estimating DOA can be obtained, but it is not what we focus on. Our aims are to recover the sparse signal matrix and achieve the high-resolution based on the designed weight matrix. Hence the coefficients of the RD-Capon spatial spectrum are exploited to design a weight matrix for achieving the high resolution. Then the coefficients of the RD-Capon spatial spectrum can be constructed, which is shown as

$$\mathbf{H} = [h(\hat{\theta}_1), h(\hat{\theta}_2), \ldots, h(\hat{\theta}_J)] = [\mathbf{H}_{(1)}, \mathbf{H}_{(2)}] \tag{17}$$

where $h(\hat{\theta}) = 1/\left(\mathbf{e}^T \mathbf{Q}(\hat{\theta})^{-1} \mathbf{e}\right)$. The weight vector $\mathbf{H}$ can be divided into two parts: $\mathbf{H} = [\mathbf{H}_{(1)}, \mathbf{H}_{(2)}]$, $\mathbf{H}_{(1)}$ is composed with coefficients of RD-Capon spatial spectrum corresponding to the possible targets and $\mathbf{H}_{(2)}$ is composed with the residual coefficients of RD-Capon spatial spectrum. Then the weight matrix can be formulated as follows

$$\hat{\mathbf{W}} = \text{diag}\left([\mathbf{H}_{(1)}, \mathbf{H}_{(2)}]\right) / \max(\mathbf{H}_{(2)}) \tag{18}$$

when the snapshots $T \rightarrow \infty$, the weight vectors $\mathbf{H}_{1,i} \rightarrow 0$ and $\mathbf{H}_{2,i} > 0$. Consequently, owing to the characteristic of the RD-Capon algorithm that the entries of $\mathbf{H}_1$ are smaller than those of $\mathbf{H}_2$, i.e. $\mathbf{H}_{1,i}^{l_2} / \max(\mathbf{H}_2) < \mathbf{H}_{2,i} / \max(\mathbf{H}_2^{l_2})$. And the weighted matrix $\hat{\mathbf{W}}$ is presented to handle the multiple measurement vectors problem, in which the weight matrix $\hat{\mathbf{W}}$ is used to achieve the idea that the entries who are more likely to be zero in recovered matrix are penalized by larger weights and the other entries are punished by small weights. Lastly, the reweighted l_1 norm constraint minimization for sparse signal recovery can be formulated as follows

$$\min_{\mathbf{S}_{SV}} \left\| \hat{\mathbf{W}} \mathbf{S}_{\hat{\theta}}^{(l_2)} \right\|_1 \quad s.t. \quad \left\| \mathbf{X}_{\mathbf{SV}} - \mathbf{A}_{\hat{\theta}} \mathbf{S}_{\hat{\theta}} \right\|_F \le \tilde{\beta} \tag{19}$$

By vectorizing the matrices in Eq. (19), we can get its alternative formulation.

$$\min_{\mathbf{S}_{SV}} \left\| \hat{\mathbf{W}} \mathbf{S}_{\hat{\theta}}^{(l_2)} \right\|_1 \quad s.t. \quad \left\| \bar{\mathbf{X}}_{\mathbf{SV}} - \bar{\mathbf{A}}_{\hat{\theta}} \bar{\mathbf{S}}_{\hat{\theta}} \right\|_F \le \tilde{\beta} \tag{20}$$

where $\bar{\mathbf{X}}_{\mathbf{SV}} = \text{vec}(\mathbf{X}_{\mathbf{SV}})$, $\bar{\mathbf{S}}_{\hat{\theta}} = \text{vec}(\mathbf{S}_{\hat{\theta}})$, and $\bar{\mathbf{A}}_{\hat{\theta}} = \text{blkdiag}(\mathbf{A}_{\hat{\theta}}, \ldots, \mathbf{A}_{\hat{\theta}})$ is an block diagonal matrix. The Eq. (20) can be efficiently calculated by SOC (second order cone) programming software packages such as SeDuMi (Sturm 1999) and CVX (Grant and Boyd 2012). Then the DOA estimates are obtained by plotting $\mathbf{S}_{\hat{\theta}}^{(l_2)}$, solved from (20).

As shown in the Eq. (20), the regularization parameter $\tilde{\beta}$ balances the fitness of the solution to data with the sparsity prior, so it is important to choose the regularization parameter $\tilde{\beta}$ properly. Generally speaking, when noise is independent and identically distributed Gaussian, $\|\mathbf{W}_{\mathbf{SV}}\|_F$ has a χ^2 distribution with $M^2 \times K$ degrees of freedom, so setting the probability $p = 0.01$ to determine the value of $\tilde{\beta}$ is good enough, where its Matlab-based calculation is via the function $chi2inv(1 - p, M^2 \times K)$.

4 Performance analysis and Cramer–Rao bound

Remark 1 As a rough rule of thumb, the weights should relate inversely to the true signal magnitudes. As shown in Eq. (18), the proposed method uses Lagrange multipliers to solve a constrained estimation problem, and then the coefficients of the RD-Capon spatial spectrum are exploited to design a weight matrix for l_1 norm minimization. Owing to having $\mathbf{S}_j^{(l_2)} \le \sqrt{\frac{1}{\mathbf{W}_j}}$, if $\mathbf{S}_j^{(l_2)}$ corresponds to the actual signal, $\hat{\mathbf{W}}_j$ is relatively small and $\hat{\mathbf{W}}_j \mathbf{S}_j^{(l_2)}$ will approach to 1. Otherwise, $\hat{\mathbf{W}}_j$ is relatively large and $\hat{\mathbf{W}}_j \mathbf{S}_j^{(l_2)}$ will approach to 0. Therefore, the weights can counteract the influence of the signal magnitude of the l_1 norm penalty function.

Remark 2 Regarding the computational complexity, the main complexity of the proposed algorithm is in constructing the weighted matrix and solving the reweighted l_1 norm constraint minimization. The formulation of the weighted matrix requires $O\left\{LM^2N^2 + M^3N^3 + J(M^3N + M^3N + M^2)\right\}$, and solving the sparse signal reconstruction process requires $O(K^3J^3)$. Thus, the computational complexity of the proposed method is $O\left\{LM^2N^2 + M^3N^3 + J(M^3N + M^3N + M^2) + K^3J^3\right\}$. While the computational complexity of the RD-Capon is $O\left\{LM^2N^2 + M^3N^3 + J(M^3N + M^3N + M^2)\right\}$, and the main computational cost of the l_1-SRACV method is also the sparse signal reconstruction process, which requires $O(K^3J^3)$. Although the computational complexity of the proposed method is higher than RD-Capon method and l_1-SRACV algorithm, the proposed algorithm can achieve high-resolution than both of them, because the weight matrix can enhance the sparsity and achieve more accurate estimation.

According to Stoica and Nehorai (1990), we derive the Cramer–Rao bound (CRB) of the DOA estimation in the monostatic MIMO radar as follows:

$$\mathrm{CRB} = \frac{\sigma^2}{2L}\left\{\mathrm{Re}\left[(\mathbf{D}^H\mathbf{\Pi}_A^{\perp}\mathbf{D}\oplus\mathbf{P}^T)\right]\right\}^{-1} \tag{21}$$

where $\oplus$ represents the Hadamard product, $\mathbf{P} = 1/L\sum_{l=1}^{L}\mathbf{s}(t_l)\mathbf{s}^H(t_l) \in \mathbf{C}^{K\times K}$, $\Pi_A^{\perp} = \mathbf{I}_{MN} - \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H \in \mathbf{C}^{MN\times MN}$, $\mathbf{D} = [\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_K] \in \mathbf{C}^{MN\times K}$, $\mathbf{d}_k = \partial(\mathbf{a}_t(\theta_k)\otimes\mathbf{a}_r(\theta_k))/\partial\theta_k$.

5 Simulation results

In this section, the performance of the proposed algorithm is investigated, and compared with RD-Capon (Zhang et al. 2012), l_1-SRACV (Yin and Chen 2011), and CRB in Eq. (21). Assumed multiple narrow-band far-field signals impinge on ULA of sensors from directions as $\theta_1 = -10°$, $\theta_2 = 5°$, $\theta_3 = 10°$, the inter-element spacing is half a wavelength and both of the transmit arrays and receive arrays are ULAs. Besides, 100 Monte Carlo trials are carried for statistical calculation. The direction grid is uniform with 0.01° sampling from −90° to 90° and the confidence interval for choosing the regularization parameter is set as 99 % in all algorithms. In the following simulations, we defined the signal-to-noise ratio (SNR) as

$$\mathrm{SNR} = 10\log_{10}\left(\|\mathbf{AS}\|_F^2 / \|\mathbf{W}\|_F^2\right). \tag{22}$$

In the paper, the root mean square error (RMSE) that evaluates the performance of DOA estimation is defined as follows:

$$\mathrm{RMSE} = \sqrt{\frac{1}{100K}\sum_{i=1}^{100}\sum_{k=1}^{K}\left(\hat{\theta}_{k,i}-\theta_k\right)^2} \tag{23}$$

where $\hat{\theta}_{k,i}$ is the estimate of DOA of the kth signal in the ith Monte Carlo trial.

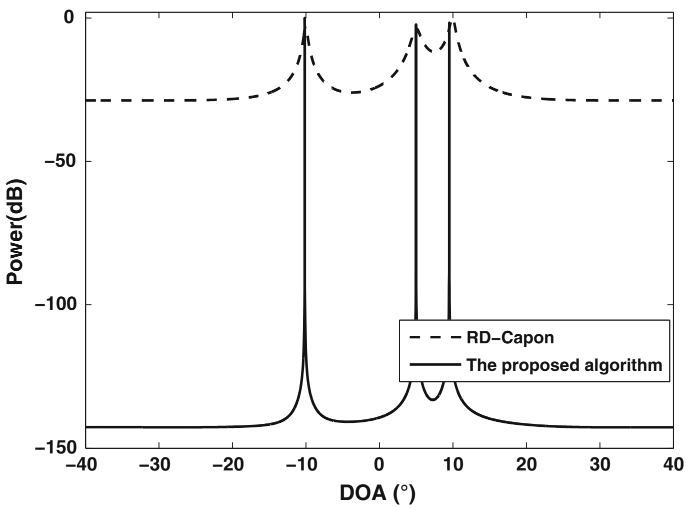

Fig. 2 Spatial spectra for RD-Capon algorithm and the proposed algorithm for uncorrelated sources

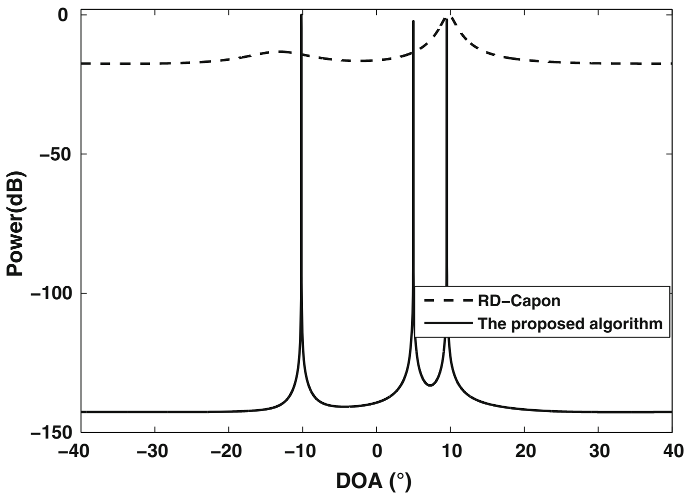

Fig. 3 Spatial spectra for RD-Capon and the proposed algorithm for coherent sources

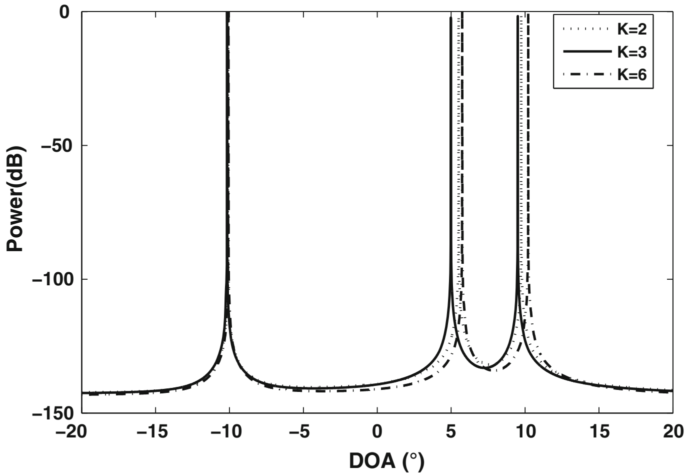

Fig. 4 Sensitivity of the proposed algorithm to the assumed number of sources

Figure 2 shows the capability of characterizing three uncorrelated sources by RD-Capon algorithm and the proposed algorithm, where $M = N = 6$, $L = 200$, SNR = 0 dB. Both RD-Capon algorithm and the proposed algorithm can resolve the three sources. Besides, the proposed algorithm can primly provide sharp peaks than RD-Capon algorithm in DOA estimation, which means that the proposed algorithm has better high resolution.

Figure 3 compares the spatial spectra obtained by RD-Capon algorithm and the proposed algorithm for three coherent sources, where $M = N = 6$, $L = 200$, SNR = 0 dB. It is shown that without requiring any decorrelation operation, the proposed algorithm is still able to resolve three coherent sources, whereas RD-Capon algorithm merge the peaks. This demonstrates the high-resolution ability of the proposed algorithm. Thus the proposed algorithm can also resolve coherent sources successfully.

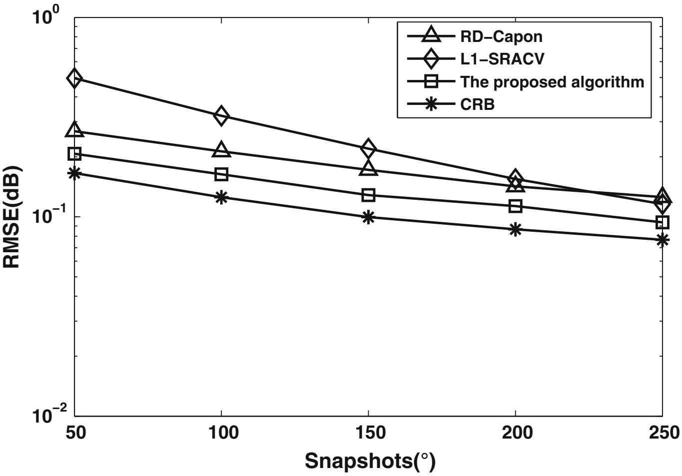

Fig. 5 RMSE of DOAs for three uncorrelated sources versus different snapshots

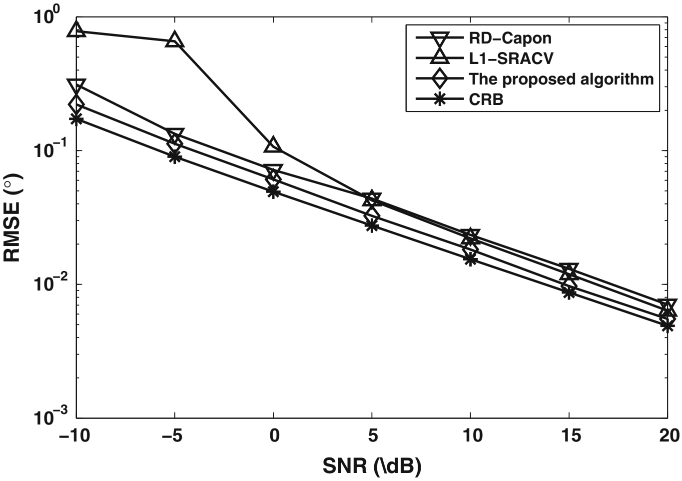

Fig. 6 RMSE of DOAs for three uncorrelated sources versus different SNRs

Figure 4 further examines the sensitivity of our algorithm to the priori knowledge of the different number of sources, where $M = N = 6$, $L = 200$, SNR = 0 dB. From Fig. 4, the proposed algorithm has the small diversity in the spectra. There are two reasons as follows: firstly, the weight matrix is used to enhance the sparsity of solution. Secondly, since the weight matrix in the proposed algorithm does not rely on the priori knowledge of the number of sources. Therefore, the proposed algorithm can achieve accurate angle estimation.

Figure 5 depicts the RMSE of DOA estimation produced by the RD-Capon algorithm, l_1-SRACV algorithm and the proposed algorithm with different snapshots, where $M = N = 8$ and SNR = 0 dB are used. As shown in Fig. 5, we can see that the proposed algorithm has better angle estimation performance than RD-Capon algorithm and l_1-SRACV algorithm with snapshots increasing. The l_1-SRACV algorithm is sensitive to small number of snap-

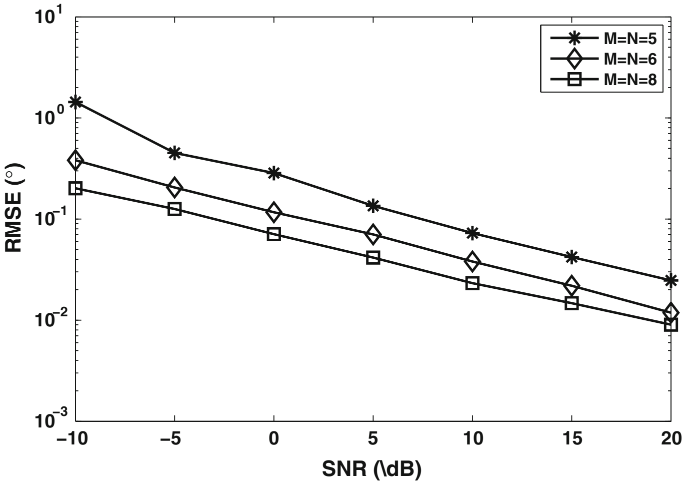

Fig. 7 RMSE of DOAs for three uncorrelated sources versus different sensors

shots, while the proposed algorithm can reduce the sensitivity to small number of snapshots effectively, because of the sparse representation model as well as the weight matrix we adopt.

Figure 6 shows the RMSE of DOA estimation via SNR with different algorithms, where $M = N = 8$ and $L = 200$. We vary the SNR from -10 to 20 dB in 5 dB steps. From Fig. 6, we can see that the proposed algorithm performs better than the other algorithms, especially in low SNR. Owing to the weighted matrix designed by the RD-Capon spatial spectrum, the proposed algorithm can enhance the sparsity of the solution effectively and be the closest to the CRB.

Figure 7 shows that the RMSE of DOA estimation produced by the proposed algorithm decreases monotonically with the number of antenna sensors increasing, where the number of snapshots and SNR are set to be 200, 0 dB, respectively. From Fig. 7, it can be verified that increasing of the number of sensors is an efficient method to improve the estimation performance.

6 Conclusion

In this paper, we have presented a reweighted l_1 norm penalty algorithm for MMV problem in monostatic MIMO radar. The SVD technique is utilized to reduce the computational complexity and the sensitivity to noise. Then a constrained minimization problem is formulated, and the coefficients of the RD-Capon spatial spectrum are exploited to design a weight matrix for reweighting l_1 norm penalty minimization. Finally, the DOAs can be obtained by solving the reweighted l_1 norm constraint minimization. Simulation results verified that the proposed algorithm can provide better angle estimation performance than RD-Capon and l_1-SRACV algorithms. In addition, the proposed algorithm is suitable for coherent sources and not sensitive to the incorrect determination of the number of sources. Furthermore, the sparsest solution of l_1 norm penalty can approximate to the l_0 norm penalty effectively, and the RMSE of the proposed algorithm is close to the CRB.

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